COMBINED SHIP HULL HORIZONTAL AND VERTICAL WAVE BENDING MOMENTS IN IRREGULAR SEAS

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ABSTRACT

by

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Rational design methods based on a statistical description of both the ship load and strength have been proposed by several authors in lieu of the traditional bending moment calculations. In general, the effect of horizontal moments on the primary strength has been neglected. It is shown in this paper, as several others have found, that the horizontal moments can be very significant. Two methods are described to obtain the magnitude of the equivalent combined moment.

An example application is shown for a large tanker by use of the MIT seakeeping program. Wave moment trends with speed, sea state and wave heading are noted. Measured and predicted values of the vertical bending stress are compared. Finally, the parameters of the weibull distribution to describe long-term wave moment amplitudes are derived.

ACKNOWLEDGEMENTS

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NOTATION

L = ship's length

H = wave height

h = significant wave height

M_V = vertical bending moment

M_H = horizontal bending moment

M_T = equivalent combined bending moment

k_V = vertical section modulus

k_H = horizontal section modulus

K = section modulus ratio

E = mean square value of the amplitude of the response

s,k = parameters of the Weibull distribution

$\bar{m}$ = mean value of the response amplitude

V_n = amplitude of the vertical moment component

H_n = amplitude of the horizontal moment component

V = ship's speed

L_e = effective wave length

RAO = response amplitude operator

RMS = root mean square

ω = circular frequency

θ = phase angle of the horizontal moment with respect to a wave with its crest amidship

ε = phase angle of the vertical moment with respect to a wave with its crest amidship

ϕ = phase angle of the equivalent combined moment

ρ_{VH} = correlation coefficient of the vertical and horizontal bending moment

σ_T = total or combined stress

σ_V = vertical bending stress

σ_H = horizontal bending stress

λ = wave length

$S_R(\omega)$ = spectral density of the response

$H(\omega)$ = RAO of the ship

$S(\omega)$ = sea spectral density

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Chapter I

INTRODUCTION

The traditional method for determining the load for primary strength calculations consists of poising the ship on a trochoidal wave of length equal to that of the ship and wave height equal to one-twentieth (1/20) of the length. The static bending moment is then determined by the load-shear-moment relations from beam theory. There have been various proposals to modify the wave height (e.g., $H = 0.6L^{0.6}$ or $H = 1.1\sqrt{L}$) partly to correct for dynamic effects and partly because it is thought than an L/20 wave is too severe for the very large ships. Notwithstanding, this empirically derived procedure was considered entirely satisfactory until relatively recently and is still in prevalent use inasmuch as it gives conservative estimates.

The appearance of large ocean-going tankers and bulk carriers and novel ship types gave impetus towards the search for a "rational" procedure for ship structural design. It was realized that the application of the standard bending moment calculations for these ships lead to unrealistic results. At the same time new knowledge relating to ship structural design had accumulated and experimental and computational facilities to advance this knowledge have improved tremendously.

Many marine-oriented societies and organizations undertook basic research to improve the accuracy of predictions of sea loads and motions, and the development of a rational design method applicable for all ship types. Full scale and model tests data were collected. The random nature of both the loads on a ship and its capacity to resist these loads was recognized, so that probability theory was utilized to develop statistical models for the design of ships [1-14]*. The advantages of a probabilistic approach over the traditional deterministic calculations are discussed in reference [7].

In the conventional method, the primary strength of a ship is based solely on the longitudinal stresses caused by vertical bending, or bending about the horizontal neutral axis. The effects of lateral bending, or bending about the vertical neutral axis, is neglected. However, for ships travelling oblique to the waves, the lateral loads reach a magnitude equal to or greater than the vertical bending moment. The combined effect of lateral and longitudinal bending may be critical for some ship types.

A requisite for an integrated probabilistic design procedure is a more precise knowledge of the loads acting on

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*The numbers in brackets refer to references listed in the Appendix

a ship. This paper represents a small portion of the total effort to completely define the structural loads experienced by a ship in a seaway. It is an investigation of the magnitude of combined horizontal and vertical bending in irregular seas. Due to their relative phase difference, the amplitudes of the vertical and horizontal moments generally are not acting at the same time.

In the analysis, the loads are treated as random variables. Consideration of the total longitudinal stress at the deck leads to the definition of an equivalent combined bending moment. Two methods are developed to generate the statistical parameters of the combined load so that the usual statistical methods to predict long-term extreme values of the load can be applied. An example application is then made for a large tanker which had been previously instrumented so that full scale data for the vertical bending stresses experienced in service are available for purposes of comparison. Such a comparison serves to check yet another time the validity of computational techniques to simulate ship motion and loads in waves, and in particular the MIT 5-D seakeeping program. The predicted moment responses are compared and analyzed for trends.

Chapter II

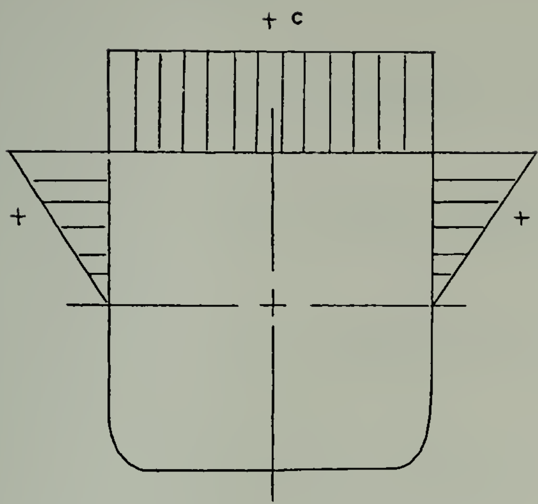
PROCEDURE

II.1 Determination of the Equivalent Combined Wave Bending Moment

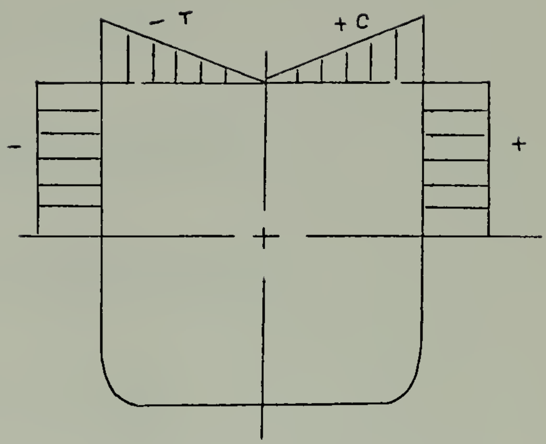
A ship travelling at an angle to the waves will be subjected to unsymmetrical bending such that the stress at the deck is not uniform but assumes a maximum value at one edge, as shown in figure II.1. This primary stress may be thought of as the combined effect of vertical and horizontal bending and, to a much lesser extent, torsion. Previous studies [15, 16] have established that torsional moments are relatively small, never exceeding 10% of the vertical moment in any given condition. In this study the effects due to torsion are considered negligible, but it is recognized that torsion loading may be critical for some ships, as those with large hatch openings [17]. The contribution to the ship primary stress may be included, if so desired, in a manner analogous to the method described here [18].

The stress at the deck edge is given by

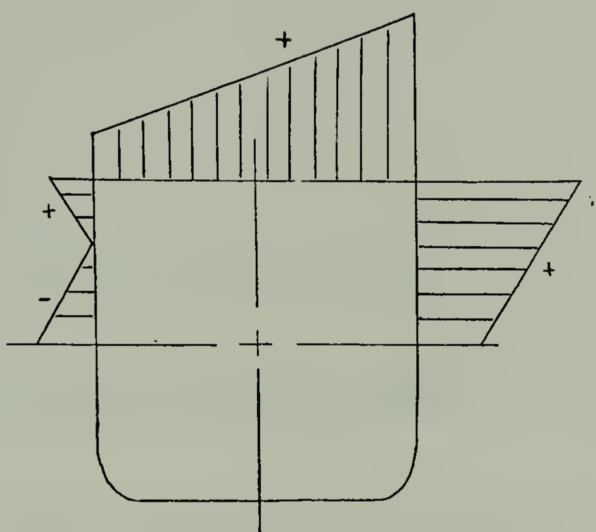
$$\sigma_T = \sigma_V + \sigma_H$$



Vertical Bending Stress
(sagging)



Horizontal Bending Stress



Combined Stress

Figure II.1

The Longitudinal Stress Distribution
on a Ship Hull Girder

where

σ_T = combined, or total, stress

σ_V = stress due to vertical bending

σ_H = stress due to horizontal bending

In terms of moments, one can write

$$\sigma_T = \frac{M_V}{k_V} + \frac{M_H}{k_H}$$

where

M_V = vertical bending moment

M_H = horizontal bending moment

$k_V = \frac{Iyy}{z}$ = vertical section modulus

$k_H = \frac{Izz}{y}$ = horizontal section modulus.

multiplying both sides of the equation by k_V, one gets

$$\sigma_T k_V = M_V + \frac{k_V}{k_H} M_H$$

If one defines an equivalent combined bending moment $M_T = \sigma_T k_V$, and the section modulus ratio $K = \frac{k_V}{k_H}$, then one gets the relation

$$M_T = M_V + KM_H \tag{1}$$

It is emphasized here that one cannot simply add the maximum values of the vertical and horizontal moments to get the maximum combined effect. In general the moments at any point along the length of the ship are not acting in phase. Furthermore, it may be of interest to specify the complete distribution of the primary stress at the deck. Within the limits of the beam theory, this is done by getting the stress at one other point across the ship's deck. A convenient reference point is at the ship's centerline where the stress due to lateral bending is zero for most ships due to port-and-starboard symmetry, i.e., $\sigma_T = \sigma_V$ at the centerline.

In the statistical approach to sea loads, the exciting force is assumed to be a stationary, narrow-band and gaussian process with zero mean over a short period of time (e.g. 30 minutes). The ship is considered to be a linear system so that the response is also stationary, narrow-band and gaussian with zero mean over the same period of time. Then the amplitudes follow the Rayleigh distribution [19, 20]. Over a long period of time, the assumption of a stationary excitation process cannot be made. However, it was found

that the amplitudes follow closely the Weibull distribution [8, 9, 10]. It is shown that the Rayleigh is a special case of the Weibull distribution given by:

$$f_X(x) = (s/k)(x/k)^{s-1} \exp -(x/k)^s \qquad X \geq 0 \qquad (2)$$

$$F_X(x) = 1 - \exp -(x/k)^s \qquad X \geq 0 \qquad (3)$$

wher $f_X(x)$ and $F_X(x)$ are the probability density and distribution functions respectively, and s and k are parameters. To get the Rayleigh distribution, one sets s=2 and $k = \sqrt{E}$ where E is the mean square value of the amplitude of the response. Furthermore it was found that the best-fit Weibull curve is approximately the exponential distribution obtained from equations (2) and (3) by setting $s = 1$ and $k = \bar{m}$ where $\bar{m}$ is the mean value of the response amplitude. The statistics of the ship bending moment response, therefore, can be specified if the mean square value, or variance, of the process is known.

The usual method to obtain the variance of the response in random seas, attributed to St. Denis and Pierson, consists of specifying the energy density spectra of the ocean waves and obtaining the ship response amplitude operator (RAO), or the response per unit amplitude of regular wave, by model tests or some computational procedure using the strip theory approach. From the theory of linear systems, the spectral density of the response can be found from the

relation [21]:

$$S_R(\omega) = |H(\omega)|^2 S(\omega)$$

where $S_R(\omega)$ = spectral density of the response

$H(\omega)$ = ship response amplitude operator (RAO)

$S(\omega)$ = spectral density of the seaway.

The area under the response spectrum is the variance of the process and represents the sum of the responses to the infinitely many sine waves that are thought to compose the actual sea.

Two methods used to obtain the variance of the wave bending moments by computer simulation techniques are explained below.

1) First Method

Equation (1) can be written showing the moments as a continuous random function of time, t:

$$M_T(t) = M_V(t) + KM_H(t)$$

Using the assumption that the sea can be represented by infinitely many harmonic waves of small amplitudes, continuous frequency and random phase, one can write the vertical and horizontal moment responses as:

$$M_V(t) = V_n \cos(\omega_n t + \varepsilon_n), \text{ and}$$

$$M_H(t) = H_n \cos(\omega_n t + \theta_n)$$

where

V_n = amplitude of the vertical moment response to the nth wave

H_n = amplitude of the horizontal moment response to nth wave

ω_n = frequency of the nth wave

ε_n = phase angle of the vertical moment with respect to a wave with its crest amidship

θ_n = phase angle of the horizontal moment with respect to a wave with its crest amidship

then

$$M_T(t) = \sum_{n=1}^{\infty} V_n \cos(\omega_n t + \varepsilon_n) + K \sum_{n=1}^{\infty} H_n \cos(\omega_n t + \theta_n)$$

or

$$M_T(t) = \sum_{n=1}^{\infty} \left[V_n \cos(\omega_n t + \varepsilon_n) + KH_n \cos(\omega_n t + \theta_n)\right]$$

which can be rewritten as

$$M_T(t) = \sum_{n=1}^{\infty} \Big[\left(V_n \cos \varepsilon_n + KH_n \cos \theta_n\right) \cos \omega_n t$$

$$- \left(V_n \sin \varepsilon_n + KH_n \sin \theta_n\right) \sin \omega_n t \Big]$$

Let

$$X_n \equiv V_n \cos \varepsilon_n + KH_n \cos \theta_n$$

$$Y_n \equiv V_n \sin \varepsilon_n + KH_n \sin \theta_n$$

Then,

$$M_T(t) = \sum_{n=1}^{\infty} \left[X_n \cos \omega_n t + Y_n \sin \omega_n t \right] \qquad (4)$$

Since $M_T(t)$ is also a continuous random function of time, it can also be represented by:

$$M_T(t) = \sum_{n=1}^{\infty} Z_n \cos\left(\omega_n t + \theta_n\right) \qquad (5)$$

A comparison of equations (4) and (5) shows that

$$\theta_n = \tan^{-1} \frac{Y_n}{X_n}, \quad \text{and} \quad Z_n^2 = X_n^2 + Y_n^2$$

Then

$$Z_n^2 = V_n^2 + K^2H_n^2 + 2KH_nV_n\cos(\epsilon_n - \theta_n) \qquad (6)$$

Equation (6) gives the amplitude of the combined moment due to vertical and horizontal bending, with the phase difference in the occurence of the component peaks taken into consideration. The MIT 5-D seakeeping program calculates the ship motions and the vertical and horizontal moment responses to regular waves (RAO). A slight modification of the program to obtain the equivalent combined moment response operator was made by use of equation (6). The RAO's were then used in the usual power spectral analysis in the frequency domain to obtain the variance of the responses in irregular seas. Comparison of predicted and experimental results have shown generally good agreement [15, 16, 21, 22, 23] and have led to widespread acceptance of the superposition method of St. Denis and Pierson.

2) Second Method

Considering the wave loads as random variables, it is generally accepted that the bending moments M_V and M_H are stationary, narrow-band, gaussian processes with zero means for short periods of time. Then M_T is also stationary, narrow-band and gaussian with zero mean, i.e., in statistical notations:

$$E\,M_T = E\,M_V + KE\,M_H = 0$$

Furthermore,

$$E\left[M_T^2\right] = E\left[\left(M_V + KM_H\right)^2\right]$$

$$E\left[M_T^2\right] = E\left[M_V^2\right] + K^2E\left[M_H^2\right] + 2KE\left[M_V \cdot M_H\right]$$

but the covariance is defined as:

$$E\left[M_V M_H\right] = \rho_{VH}\left\{E\left[\left(M_V - \bar{m}_V\right)^2\right]E\left[\left(M_H - \bar{m}_H\right)^2\right]\right\}^{\frac{1}{2}}$$

where $\bar{m}_V = \bar{m}_H$ = mean values = 0

ρ_{VH} = correlation coefficient of the vertical and horizontal bending moments

therefore,

$$E\left[M_T^2\right] = E\left[M_V^2\right] + K^2E\left[M_H^2\right] +$$

$$2K\rho_{VH}\left(E\left[M_V^2\right]E\left[M_H^2\right]\right)^{\frac{1}{2}} \qquad (7)$$

For a zero-mean process, the mean square is equal to the variance.

As in the first method, the value of the section modulus ratio, K, must be known for a given ship or estimated during preliminary design. An attempt was made to examine the section modulus ratio of several sizes of tankers to determine if there was a trend with the growth in the size of these ships. As shown in figure II.2, the points are widely scattered with no apparent trend. Since the value of K depends on the construction of the ship (framing system, scantlings, etc), perhaps the scatter of points is indicative of the present uncertainty about both sea loads and the strength of ships, and the rather arbitrary way a "safety factor" is arrived at.

Likewise, the value of the correlation coefficient, ρ_{VH}, of the vertical and horizontal moments must be determined. Implicit in the use of equation (7) is the assumption that an average value of ρ_{VH} already takes into account the phase difference of the peaks of M_V and M_H in all conditions at sea.

With this method, the bending moment response amplitude operators may be determined by experiment or computer techniques. Spectral analysis could then be performed to obtain the variances of M_V and M_H, and finally the combined moment M_T.

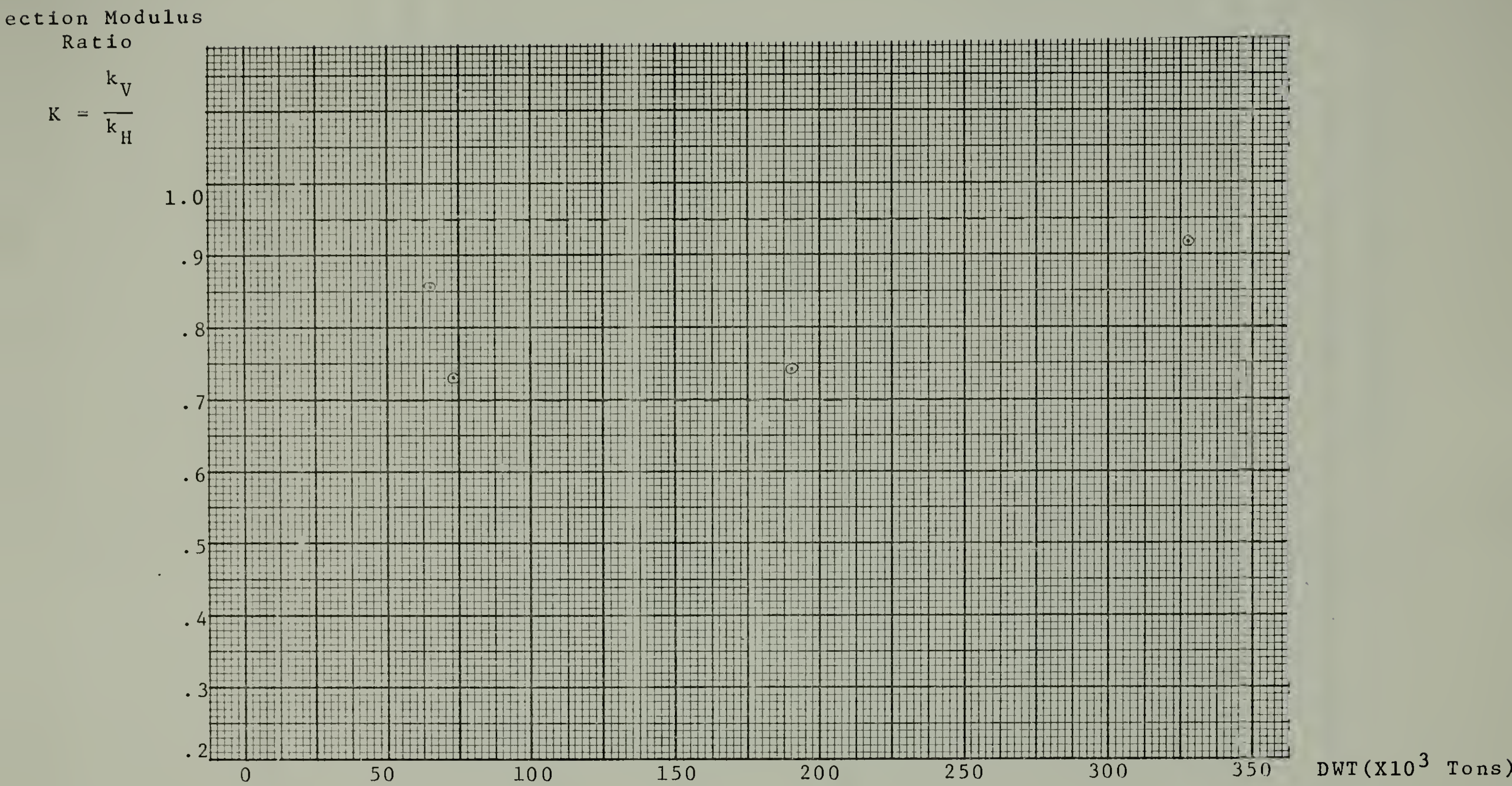


Figure II.2

Section Modulus Ratio VS DWT of Tankers

II.2 Long-Term Wave Load Analysis

After the statistics of the short-term response are determined for the range of sea states that make up the actual seaway, the expected long-term loads may be predicted. Several methods to obtain the long-term distribution of wave loads and the prediction of extreme values have been proposed [6, 9, 11, 18, 23]. Mansour [18] has described procedures for the analysis of failure under extreme loads under two conditions:

1. Short-term analysis - when the route of the ship is not known and a "severest condition" criterion is imposed.

2. Long-term analysis - If the ship's route is known and if that route is more or less permanent.

In this study, a procedure is discussed to determine the value of the parameter $k = \bar{m}$ of equations (2) and (3) for a ship travelling oblique to waves in all sea conditions described by the significant wave height and period (h,T) as in [24]. The following assumptions are made:

1. wave headings are uniformly distributed over all directions.

2. The speed is essentially determined by the state of the sea. The speed-significant wave height relation is adopted from [14] as follows:

Speed, V knots	significant wave height, h ft.
16	less than 12.5
12	12.5 to 21.5
6	greater than 21.5

The procedure followed is described below:

1. Obtain the RMS response to all sea states (i.e., range of sea spectra) in all conditions of speed and wave headings by some analytical method as the "MIT 5-D seakeeping program."

2. For each speed, get the average of the RMS response over all wave directions.

3. Use Bretschneider's two - parameter family of sea spectra to get the response to any sea described by a particular (h,T) pair. This is most easily done by scaling the responses to the sea spectra of step (1). We call this matrix of the response $\underline{R}$ with elements r_{ij}.

4. For each wave height in [24], determine the probability (i.e., relative frequency of occurence)

of each observed period. We call the resulting matrix of probabilities $\underline{P}$ with elements p_{ij}.

5. Determine the average RMS response for each significant height by the relation

$$m_i = \sum_j r_{ij} \times p_{ij}$$

where the subscripts i and j refer to rows and columns of a matrix. The resulting column matrix is called $\underline{M}$.

6. Determine the probability of occurrence of each significant wave height. We call this column matrix $\underline{Q}$ with elements q_i.

7. The mean or expected value of the RMS of the response for a particular area is given by

$$\text{mean RMS} = \sum_i m_i \times q_i$$

Then the desired parameter $\bar{m}$ = 1.25 x (mean RMS of the response) in the case of a short-term analysis [18]. For the long-term analysis, a time factor is additionally introduced to get the

weighted mean of the RMS of the response for all areas in the ship's route.

Steps 4-6 is equivalent to getting the probability of occurrence of each (h,T) pair in any one area being described. If we call this matrix $\underline{P}'$ with elements p'_{ij}, then

$$\text{mean RMS} = \sum_i \sum_j r_{ij} \times p'_{ij}$$

The data in [24] have been computerized and could be used in the above calculations.

II.3 Details of the Procedure

The methods for wave load analysis and prediction, as explained in the foregoing sections of this chapter, were applied in the analysis of a large tanker. The UNIVERSE IRELAND was one of five vessels of different sizes instrumented for full-scale measurement of stresses experienced in actual service. However, the strain gage arrangement on deck was such that the stress due to lateral bending was cancelled out, so that only vertical bending stresses were recorded. Therefore, the results derived for the response of primary interest, the combined moment, could not be verified. The description and analysis of the results of the 3½-year project sponsored by the American Bureau of Shipping (ABS) is reported in [25] and provide the basis for comparison with the analytically derived values of vertical bending stress at amidship.

The general characteristics of the UNIVERSE IRELAND are listed below:

Full load displacement, tons	375,811.
Length overall, ft	1135.17
Length between perpendiculars, ft . . .	1083.
Breadth, ft	174.87
Depth, ft	105.
Design draft (keel), ft	81-0-5
Block coefficient (L_{wl})	0.86
Section modulus, top, in^2-ft	566,794.
Section modulus, side, in^2-ft	617,350.

The ship's lines and other data pertaining to the ship necessary to generate the input for the computer analysis

were obtained from the ABS. It was necessary to use an approximate lightship weight distribution because of an apparent discrepancy in the distribution curve provided. Since the still water bending moment, at least, is very sensitive to the distribution of the weight, a probable error was anticipated in the calculated values. For example, the maximum still water bending moment calculated by the computer exceeded the value determined by the ABS by about 11.5%.

The MIT 5-D seakeeping program is fully described in [26]. Only minor modifications were necessary to do the quasi-static combined bending moment analysis. The response to long-crested irregular seas were used in conjunction with an auxiliary program to obtain the responses to short-crested seas. This program uses the cosine-squared spreading function

$$s(\mu) = \frac{2}{\pi} \cos^2(\mu)$$

where μ = angle between the wind direction and the wave components. The principal wind directions were given exactly the same values as the wave-to-course headings.

The computer was run for ship's speed of 6, 12 and 16 knots, nine wave headings from 0° to 180° in increments of

22.5^{0}, and nine sea states described by the significant height and period. The computer output consisted of the root mean square of the response, the significant height ($H^{1/3}$) and the average of the 1/10th highest ($H^{1/10}$) of the response measured peak-to-peak for all combinations of the three speeds, nine wave directions and nine sea states. Since it was desired to compare the values of the vertical, horizontal and combined wave moments, output was obtained for all three responses. Furthermore, to be as accurate as possible, the three responses were calculated for twenty stations specified along the length of the ship, even if eventually only the responses amidships were utilized in the analysis.

The sea data were carefully chosen to match the actual seaway experienced by the ship during the instrumentation period. This was done by obtaining sea data from [24] for the areas lying along the service route of the UNIVERSE IRELAND, which made regular runs between the Persian Gulf and Western Europe by way of the cape of Good Hope [25]. Although the seakeeping program can accept and use an actual sea spectrum or simulate developing or decaying seas by the Bretschneider two-parameter spectrum, the Pierson-Moskowitz one-parameter family for fully developed seas was chosen. Nine sea states with significant heights ranging from 3.6 feet to 43.3 feet (corresponding to observed periods of 1.5 seconds

to 18.5 seconds) were used. Spectral ordinates for forty frequencies were calculated while twenty five points were used to describe the response amplitude operators covering the wave frequency range $0.1 \leq \frac{\lambda}{L} \leq 9.0$ with most of the points spaced closely around the expected spectral peak at $\frac{\lambda}{L} = 1.0$. λ is the wavelength and L is the length of the ship. The response to regular waves (RAO's) were not printed in order to save on the cost of running the computer.

Chapter III

RESULTS

The following general areas of study were explored:

1. Comparison of the derived results with those in other published works on the subject.

2. Comparison of the variance of the combined moment obtained from the two methods described in the previous chapter.

3. Comparison of theoretical calculations for the vertical bending stress with full-scale measurements.

4. Examination of bending moments trends and wave load prediction.

The results of the above studies are presented in the following sections.

III.1 Comparison of Results with Other Published Works

The calculated value of the root-mean-square (RMS) of the bending moment responses are plotted in figures III.1 to III.4 as a function of the wave headings. For contrast this was done for the responses to long-crested and short-crested fully developed seas with significant wave heights of 7.4 ft., 19.2 ft. and 43.3 ft. It was intended to show the response to a low, moderate and high sea states. Since only the value of the horizontal moment in "realistic" seas are of interest, the RMS of the response were plotted only for short-crested seas. The graphs were prepared for a representative ship's speed. The general behavior of the response did not change with speed.

Faltinsen [22] describes the comparison between theory and experiments of the wave-induced horizontal and vertical forces and moments of a series 60 hull form with a block coefficient of 0.80 and a Froude member of 0.15 moving in regular waves with different headings at an angle to the direction of the waves. The predictions made by the new strip theory developed by Salvesen, Tuck, and Faltinsen, which is used in the MIT 5-D seakeeping program proved very satisfactory. The theory is described by Salvesen et al, in [27]. For vertical bending moments, the theory predicted higher values than the experiments for $\frac{\lambda}{L} < 0.7$ in quartering and following waves and lower values for $\lambda/L < 0.6$ in bow waves. This trend may also be observed by comparing the RMS

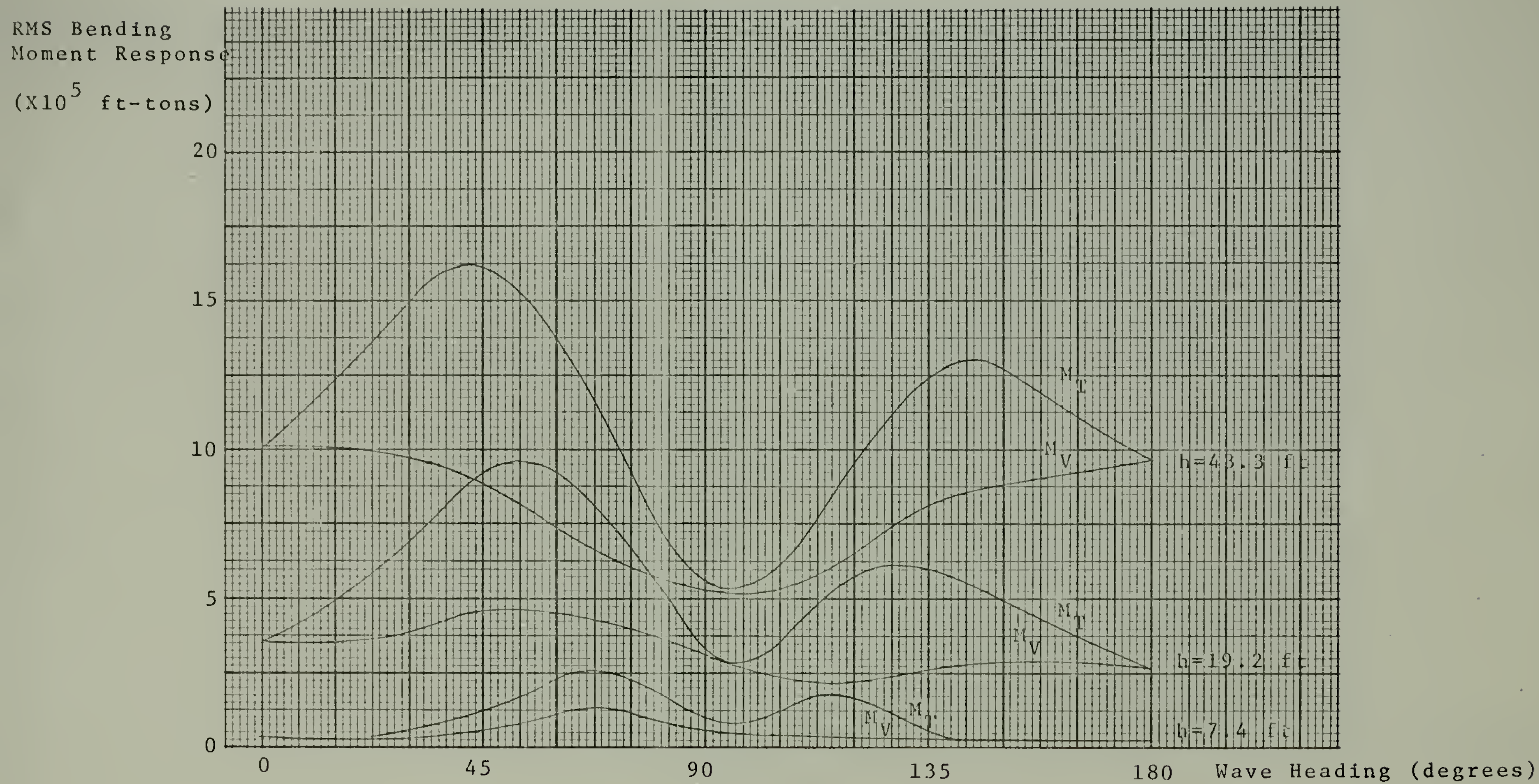


Figure III.1

RMS of the Bending Moment Response at Amidship
VS Heading in Long-Crested Waves

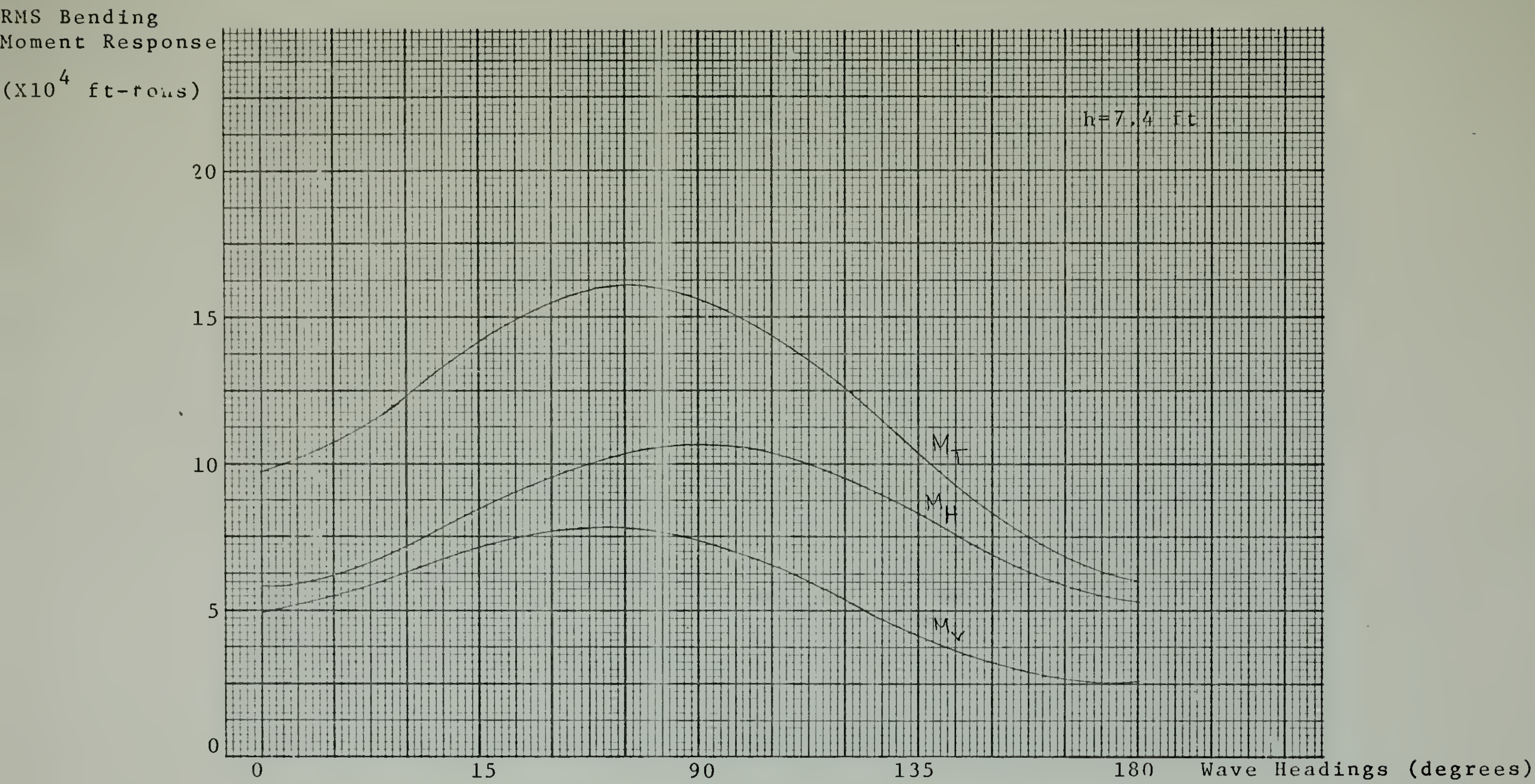


Figure III.2

RMS of the Bending Moment Response at Amidship VS Heading in Short-Crested Seas for Low Seas

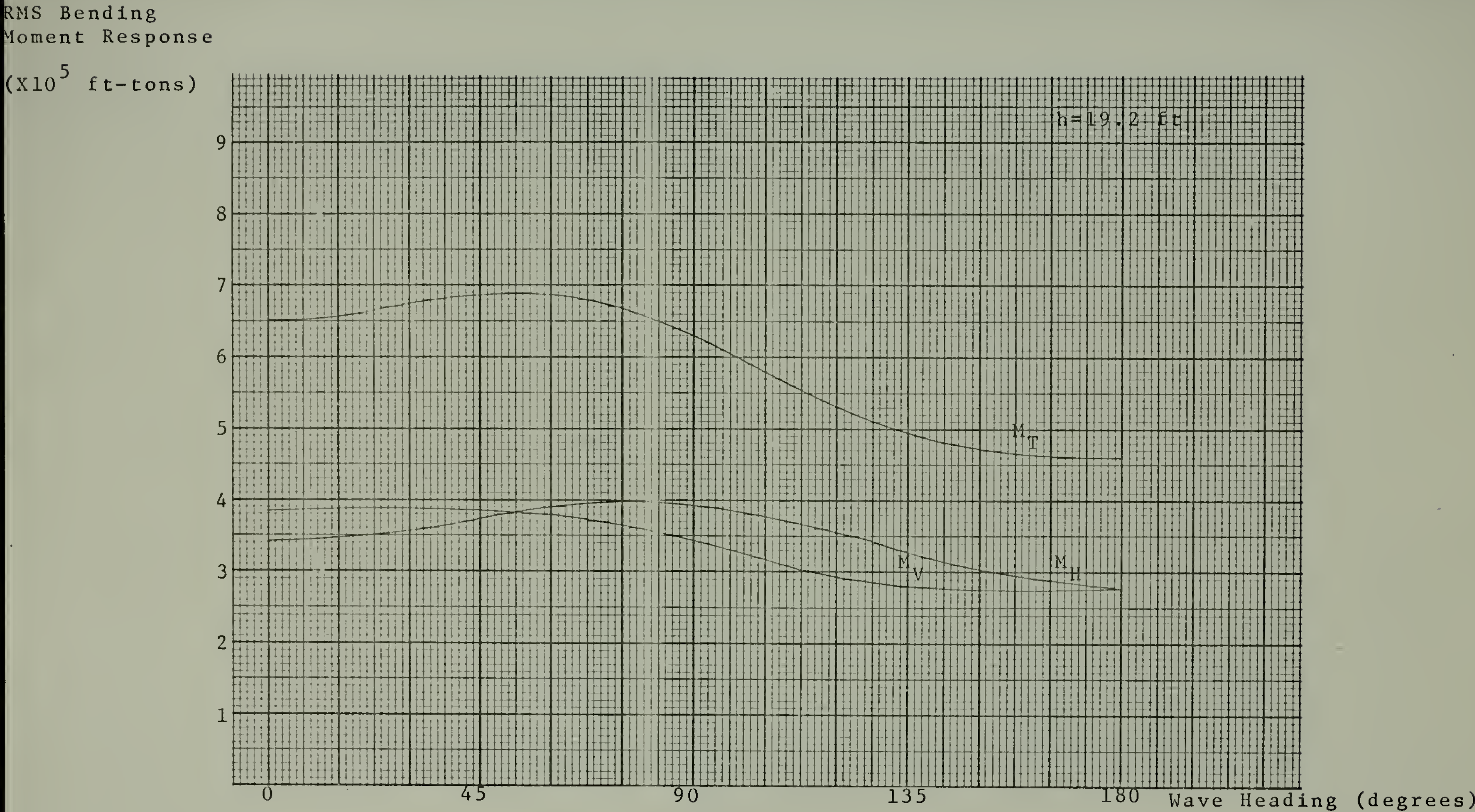


Figure III.3

RMS of the Bending Moment Response at Amidship VS Heading
in Short-Crested Waves for Moderate Seas

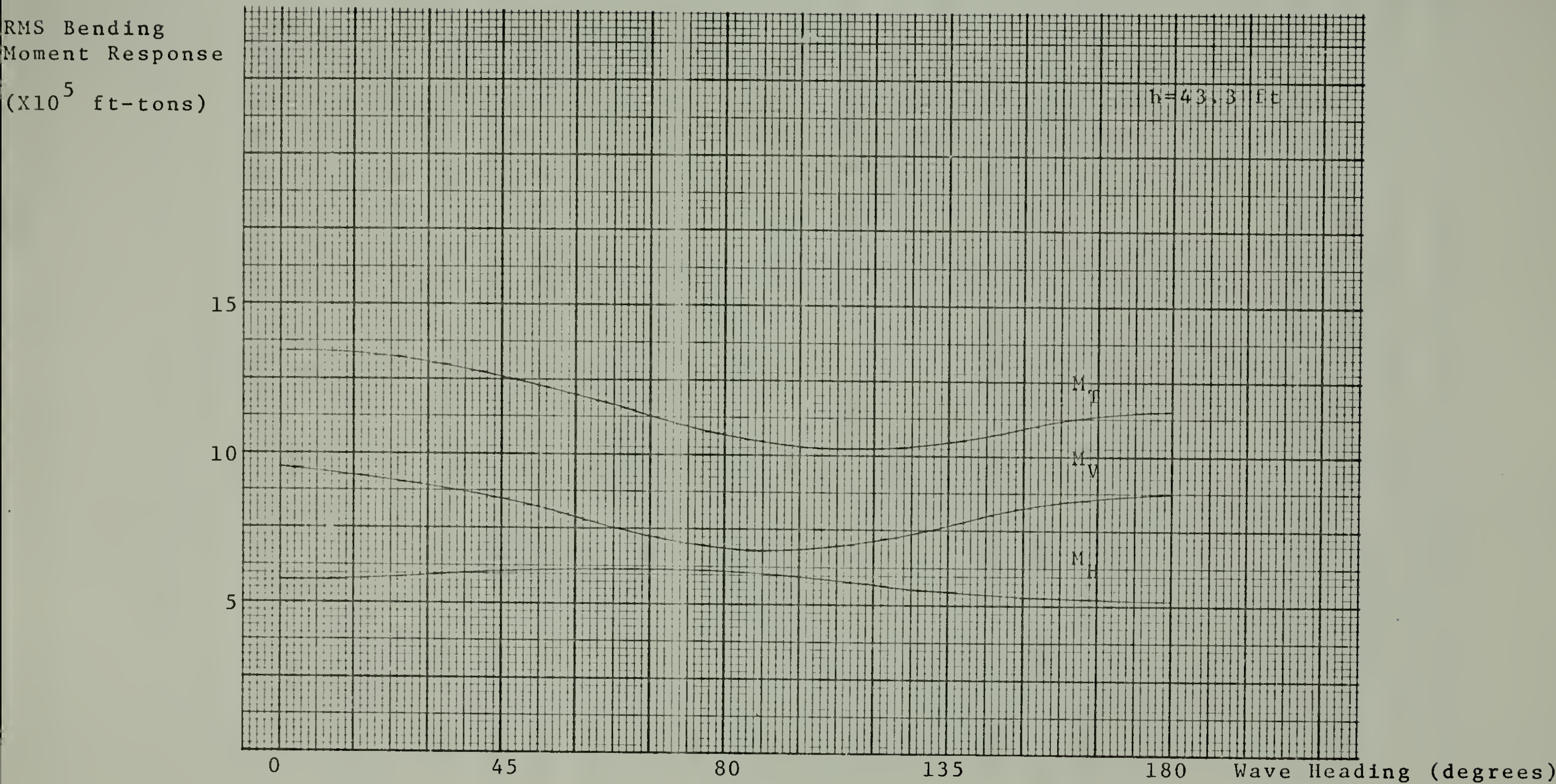


Figure III.4

RMS of the Bending Moment Response at Amidship VS Heading in Short-Crested Waves for High Seas

values of M_V in short-crested seas for wave headings of 0^o (following seas) and 180^o (head seas) in figures III.2, 3 and 4. For horizontal bending moments, appreciable difference existed between theory and experiments only for beam seas.

One of the early investigations of the effect of horizontal moments in oblique seas is described by Numata [16]. It was established in this study that lateral moments reach values that are substantial and that peak bending moment exists in the region of the speed for synchronous pitching and heaving motions. Wahab [15] shows that the importance of the horizontal bending moment increases with increasing ship length. For the sea condition used in his investigation, the significant midship horizontal moment for a ship of 200 meter-length was 46 per cent of the significant vertical moment while for a 400 meter-length ship the percentage increased to 69. The concept of effective wavelength, L_e, described by Numata was helpful to explain the results obtained from the computer analysis. However, only one sea spectrum was used in his study and some conclusions were reached that are not necessarily true for all conditions at sea for any given ship. Inasmuch as the RAO of the UNIVERSE IRELAND was not recorded by the computer, the figures in [15] proved very useful. The ship studied in that work is the same as the model described by Faltinsen and has general characteristics similar to the UNIVERSE IRELAND. The response

of the model to regular waves could be assumed to be substantially identical in shape and trends. The calculation of amidship forces and bending moments in [15] also made use of a single spectrum representing a severe sea.

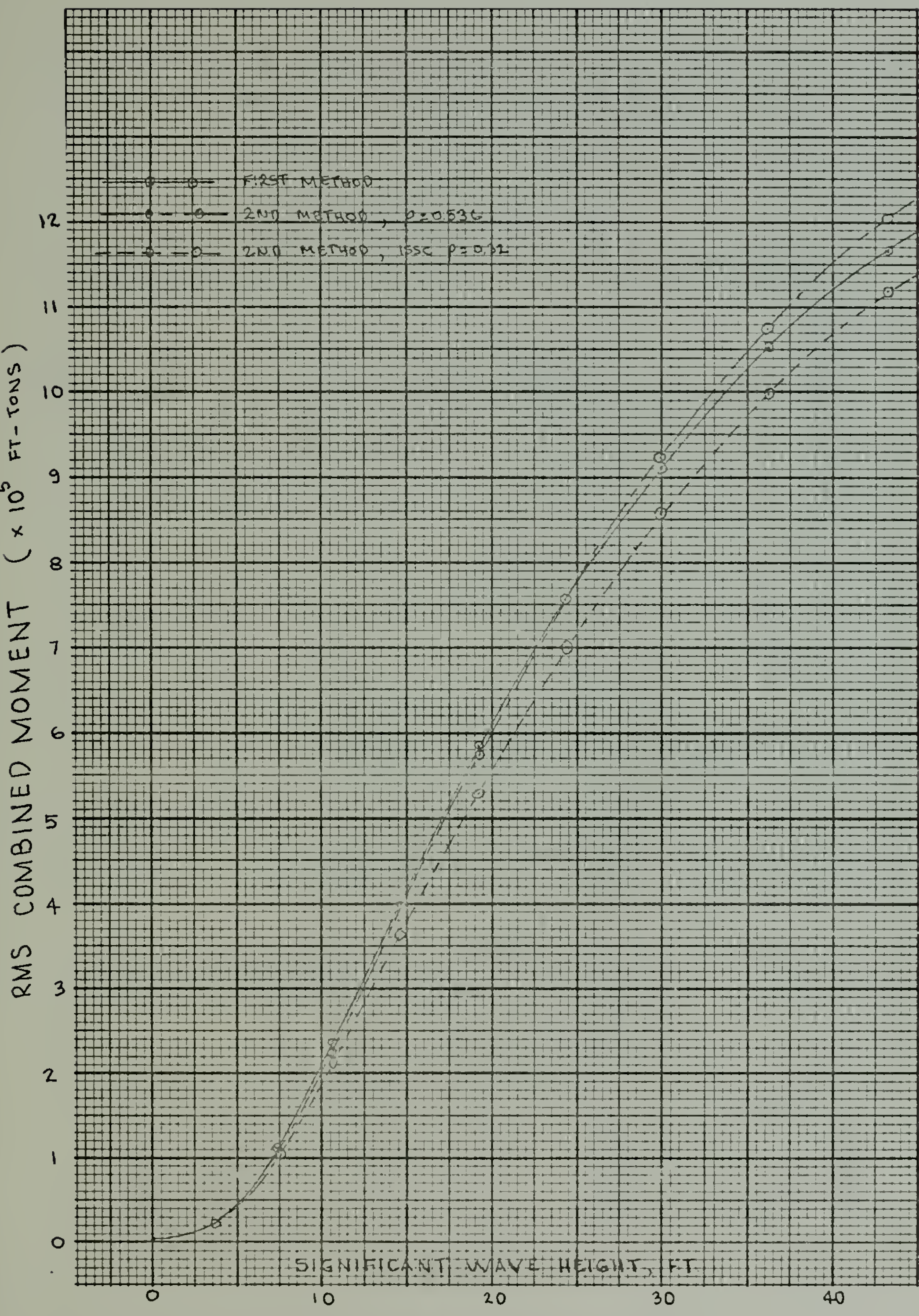
III.2 Comparison of the Calculated Values of the Combined Moment Response

Table III-1 is a summary of the results obtained for the RMS value of the combined moment response for the vessels' service speeds of 16 and 6 knots. The first set of values are those calculated from the RAO developed by the computer by use of equation (6) of the first method. The second set of values were calculated from equation (7) of the second method using the value $\rho_{VH} = 0.32$ obtained from [18] as reprinted from the published result of a Det Norske Veritas Research Report. The reprinted source is shown as figure III.5. The values of ρ_{VH} represent the correlation coefficient of the most probable largest values of the vertical and horizontal moment for one ship life of about twenty years in the North Atlantic. They were meant to be reference values and not for use in design. The difference in M_T values is shown for each sea state. It is immediately apparent that the results obtained by both methods are not very different. The third set of values was calculated also by use of equation (7) but using a value of ρ_{VH} obtained by averaging the responses in short-crested seas by the first method for all wind-wave directions and for three representative spectra for low, moderate and high seas. The mean value of ρ_{VH} obtained in this manner was 0.532, significantly higher than the ISSC value. The difference in the results for M_T is also indicated for comparison.

Table III.1

Comparison of the RMS Response for the Combined Moment at Amidships as Calculated by the Two Methods for Two Speeds

Velocity (knots)	Significant Height (ft)	1st Method RMS Response	2nd Method RMS Response (ISSC, ρ_{VH}=0.32)	% difference	2nd Method RMS Response (ρ_{VH}=0.536)	% difference
V=16	3.6	26763	27091	+1.2%	29113	+8.8%
	7.4	115468	104024	-9.9	111951	-3.0
	10.7	240620	213556	-11.2	230044	-4.4
	14.6	405516	361705	-10.8	390090	-3.8
	19.2	586565	530581	-9.5	572308	-2.4
	24.3	756493	695879	-8.0	750088	-0.8
	30.0	911865	848872	-6.9	913872	-0.2
	36.4	1050906	988128	-6.0	1062235	+1.1
	43.3	1167628	1,201,646	+2.9	1286184	+10.0
V=6	3.6	22238	21159	-4.8	22694	+2.0
	7.4	112552	99455	-11.6	106928	-5.0
	10.7	236044	208225	-11.8	224172	-5.0
	14.6	398461	371193	-6.8	397994	-0.1
	19.2	580125	529671	-8.7	571350	-1.5
	24.3	756008	700822	-7.3	755840	0.0
	30.0	913821	856749	-6.2	923295	+1.0
	36.4	1056466	998556	-5.5	1074976	+1.8
	43.3	1175724	1118861	-4.8	1203121	+2.3



Comparison of the RMS Midship Combined Moment Response as Computed by Two Methods

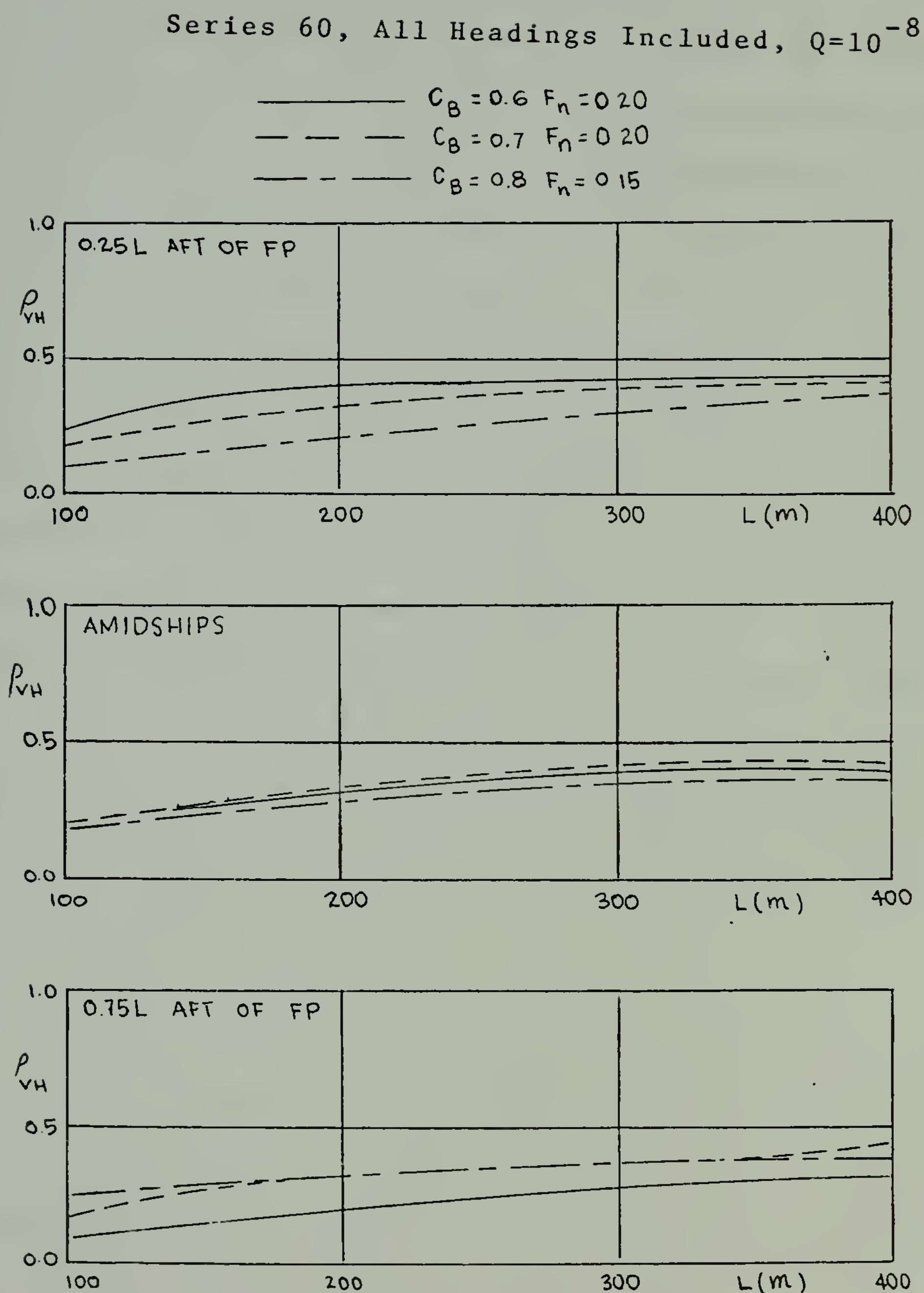


Figure III.5

Correlation Coefficient Between Vertical Bending Moment and Horizontal Bending Moment

III.3 Comparison of the Calculated Vertical Bending Stress and Full-scale measurements:

Strain gages were installed on the port and starboard sides at amidship and several other stations along the length of the UNIVERSE IRELAND to measure stresses on the deck. The arrangement was such that only vertical bending stresses were measured. Therefore, though the combined moment could not be verified, a comparison between full scale data and the computer-derived values was possible. This comparison by weather group as specified by wind measurements (Beaufort scale) is shown in figure III.6. A summary is also presented in table III.2. The conversion from bending moment to stress is done by the relation: Bending Moment = Stress x Section Modulus. The values compared are the average RMS values of the peak-to-peak stress for each wind scale. The full scale data were read off figure 11 of [25]. The conversion from wind to wave was done by the graphical relation shown in figure III.7 [23]. This conversion is summarized in table III.3.

The use of nine sea spectra with each significant wave height enables the calculation of the standard deviation of the predicted values of the vertical stress. For comparison with the measured data, these standard deviations had to be modified to obtain values based on wind measurements. This was accomplished by the relation [23]:

$$S_1^{\ 2} = S_2^{\ 2} + \left| S_3^{\ 2} - \overline{\frac{\Delta H}{2}}^2 \right| \tan^2\theta$$

where S_1 = standard deviation of the response with respect to the wind

S_2 = standard deviation of the response with respect to waves

S_3 = standard deviation of the wave height with respect to the wind, as shown in figure III.8 from [23].

ΔH = wave height increment

θ = average slope of the response vs. significant height plotted in figure III.9.

For comparison, table III.4 of the standard deviations for all the cases studied is drawn up.

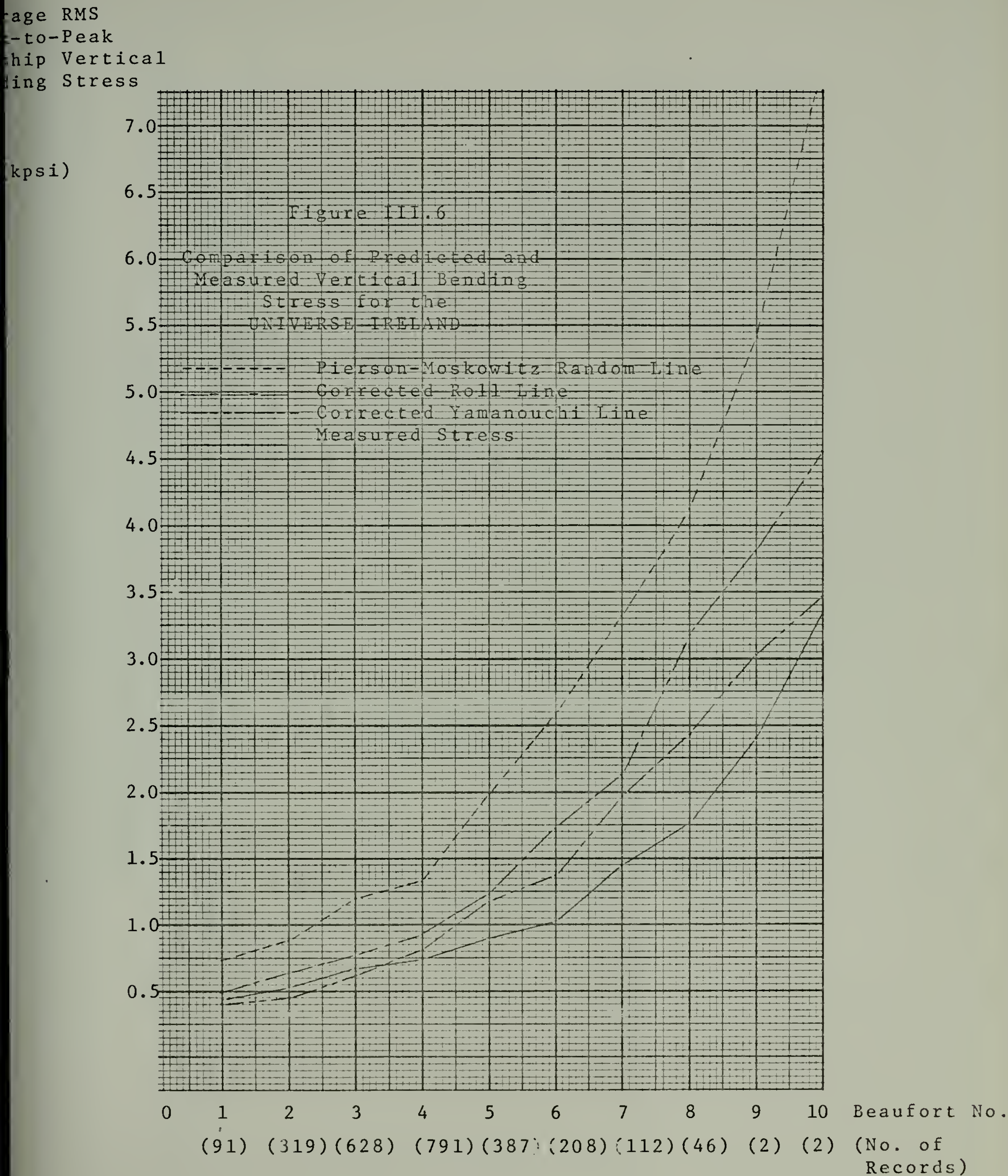


Figure III.6

Comparison of Predicted and Measured Vertical Bending Stress for the UNIVERSE IRELAND

Table III.2

Comparison of Predicted Average RMS Peak-to-Trough Vertical Bending Stress With Measured Values

Beaufort No.	Measured (kpsi)	Pierson-Moskowitz (kpsi)	Roll Modified(N.Atlantic) (kpsi)	Yamanouchi Corrected(N.Pacific) (kpsi)
1	0.450	0.736	0.486	0.419
2	0.525	0.888	0.626	0.459
3	0.675	1.206	0.777	0.626
4	0.750	1.326	0.908	0.806
5	0.900	1.992	1.206	1.181
6	1.025	2.578	1.739	1.374
7	1.462	3.314	2.140	1.991
8	1.762	4.124	3.175	2.430
9	2.400	5.346	3.812	3.042
10	3.375	7.922	4.572	3.477

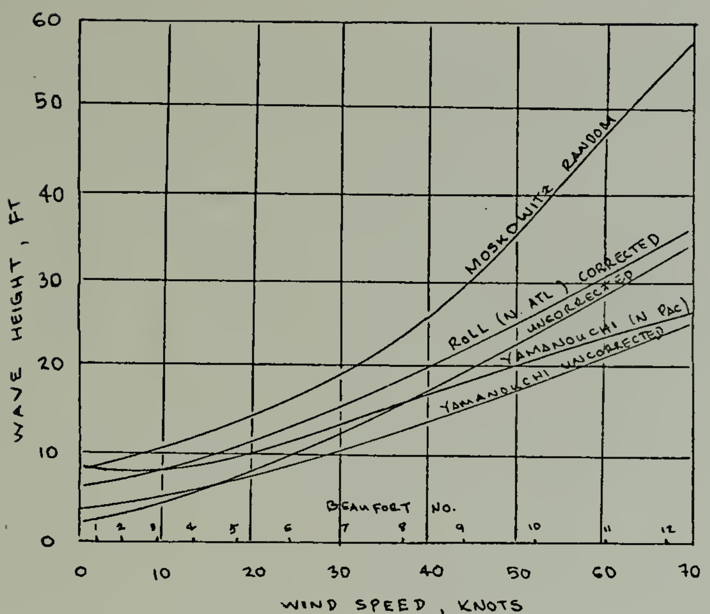


Figure III.7

Relationship Between Significant Wave Height and Wind Speed from Various Sources

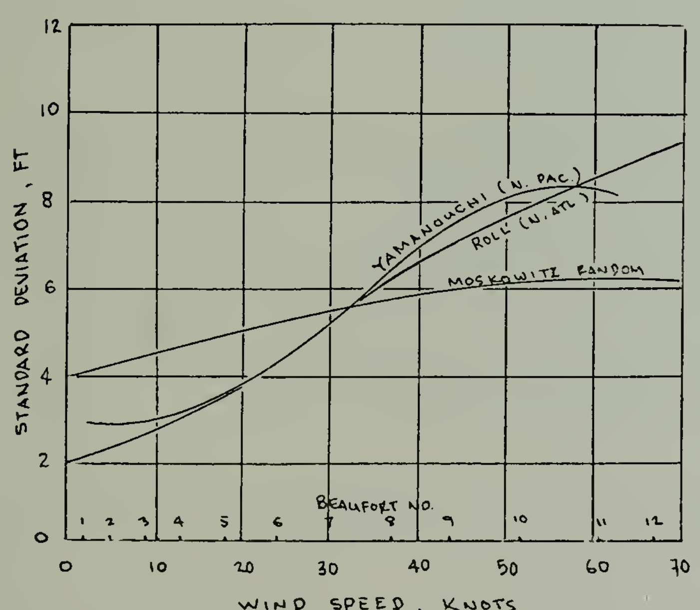


Figure III.8

Wind-Wave Height Standard Deviations from Various Sources

Table III.3

Summary of Wind-to-Wave Conversion

Beaufort No.	Pierson-Moskowitz Rand.		Roll Modified (N. Atl.)		Yamanouchi (N. Pac.)	
	mean height	std. dev.	mean height	std. dev.	mean height	std. dev.
	(ft.)	(ft.)	(ft.)	(ft.)	(ft.)	(ft.)
1	8.0	3.9	7.2	2.48	6.2	2.0
2	8.8	4.1	7.6	2.5	6.8	2.0
3	10.2	4.3	8.0	2.6	7.6	2.4
4	11.0	4.5	9.0	2.8	8.3	2.8
5	13.2	4.8	10.0	3.4	9.8	3.2
6	16.0	5.0	12.4	4.3	11.4	4.2
7	19.0	5.3	15.0	5.1	13.2	5.3
8	23.6	5.5	18.2	6.1	15.2	6.4
9	29.7	5.8	22.0	7.0	17.8	7.4
10	38.0	5.9	26.0	7.8	20.2	8.1

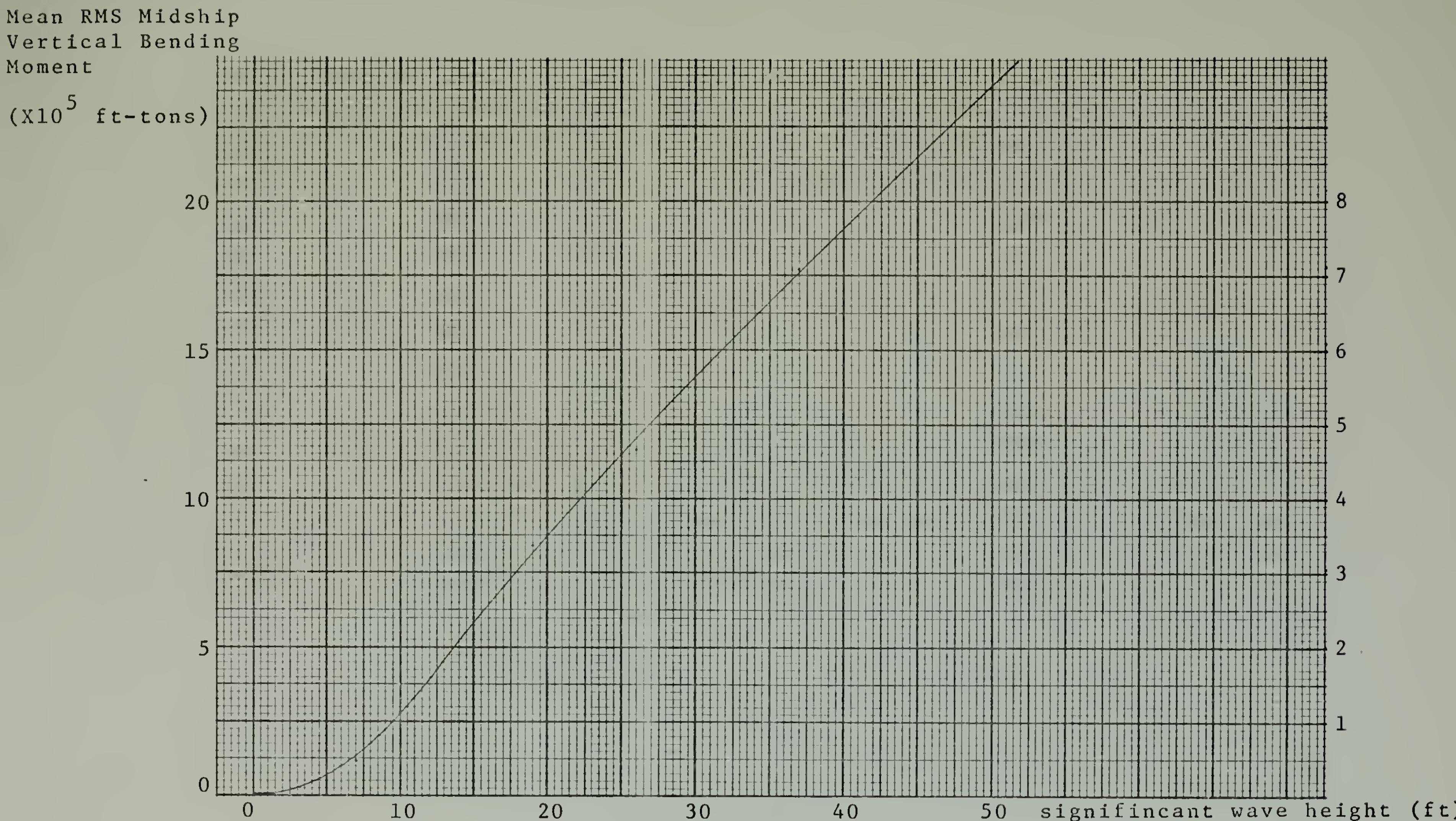


Figure III.9

Mean RMS of the Vertical Bending Response VS Significant Wave Height for all Areas in the Ship's Route

Table III.4

Standard Deviation of the Vertical Bending Stress with Respect to the Wind

Beaufort No.	Measured (kpsi)	Pierson-Moskowitz Rand. (kpsi)	Roll Modified (kpsi)	Yamanouchi (kpsi)
1	0.400	0.550	0.374	.309
2	0.400	0.549	.414	.322
3	0.425	0.604	.435	.405
4	0.475	0.639	.430	.457
5	0.500	0.738	.516	.498
6	0.500	0.938	.668	.621
7	0.625	0.973	.758	.786
8	1.000	1.134	0.997	.935
9	0.575	1.489	1.225	1.145
10	0.500	1.421	1.403	1.251

III.4 Examination of Bending Moment Trends and Wave Load Prediction

Since the traditional primary strength calculations use the vertical bending moment in a long-crested sea of arbirtary height as a basis for ship load, it is of interest to compare the values of the vertical bending response in long-crested head seas with the combined moment in short-crested seas for the higher range of the wave significant heights. Figures III.10 and III.11 are drawn for sea spectra representing fully developed moderate and severe seas, respectively moderate seas, the combined moment for head seas exceeded the long-crested vertical moment by about thirty sever per cent (37%) and in severe seas, the difference was sixteen per cent (16%).

Similarly, it was deemed desirable to compare the relative values of the vertical and horizontal moments for all conditions at sea. This is done in figures III 2, 4 and 6 for the responses in short-crested seas. One can see that the horizontal moment is not small compared to vertical moment; and in low to moderate seas, the former is at least as great as the latter. The above mentioned comparison of the response in long-crested and short-crested head seas reflect the relative contributions of the vertical and horizontal moments to the total load.

Table III.5 illustrates the change with speed in the bending moment response for low, moderate and high seas. The tabulated values are the mean RMS peak-to-mean responses in short-crested seas averaged over all wind-wave directions.

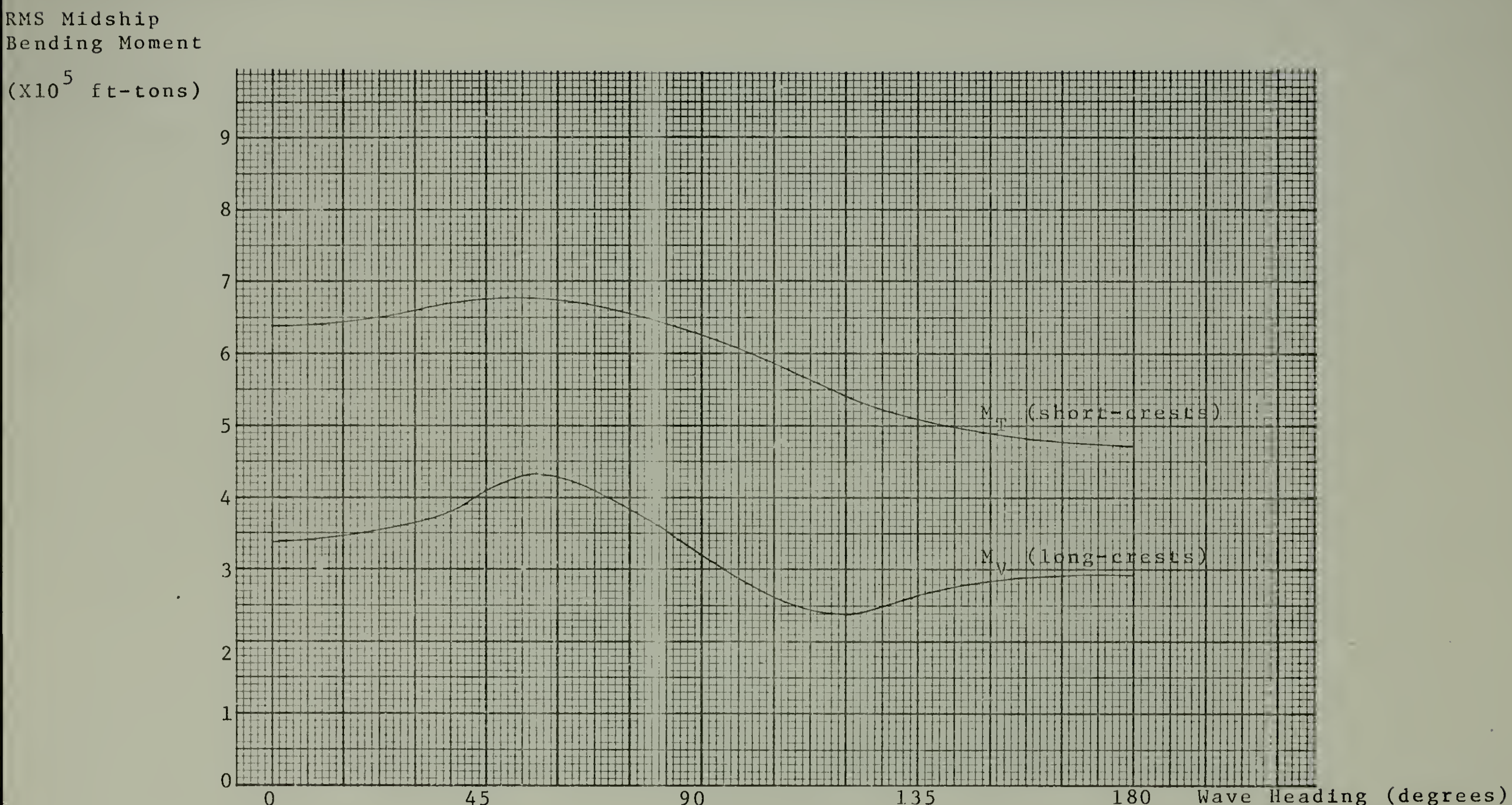


Figure III.10

Comparison of Vertical Moment Response in Long-Crested Seas VS the Combined Moment in Short-Crested Seas for h=19.2 ft

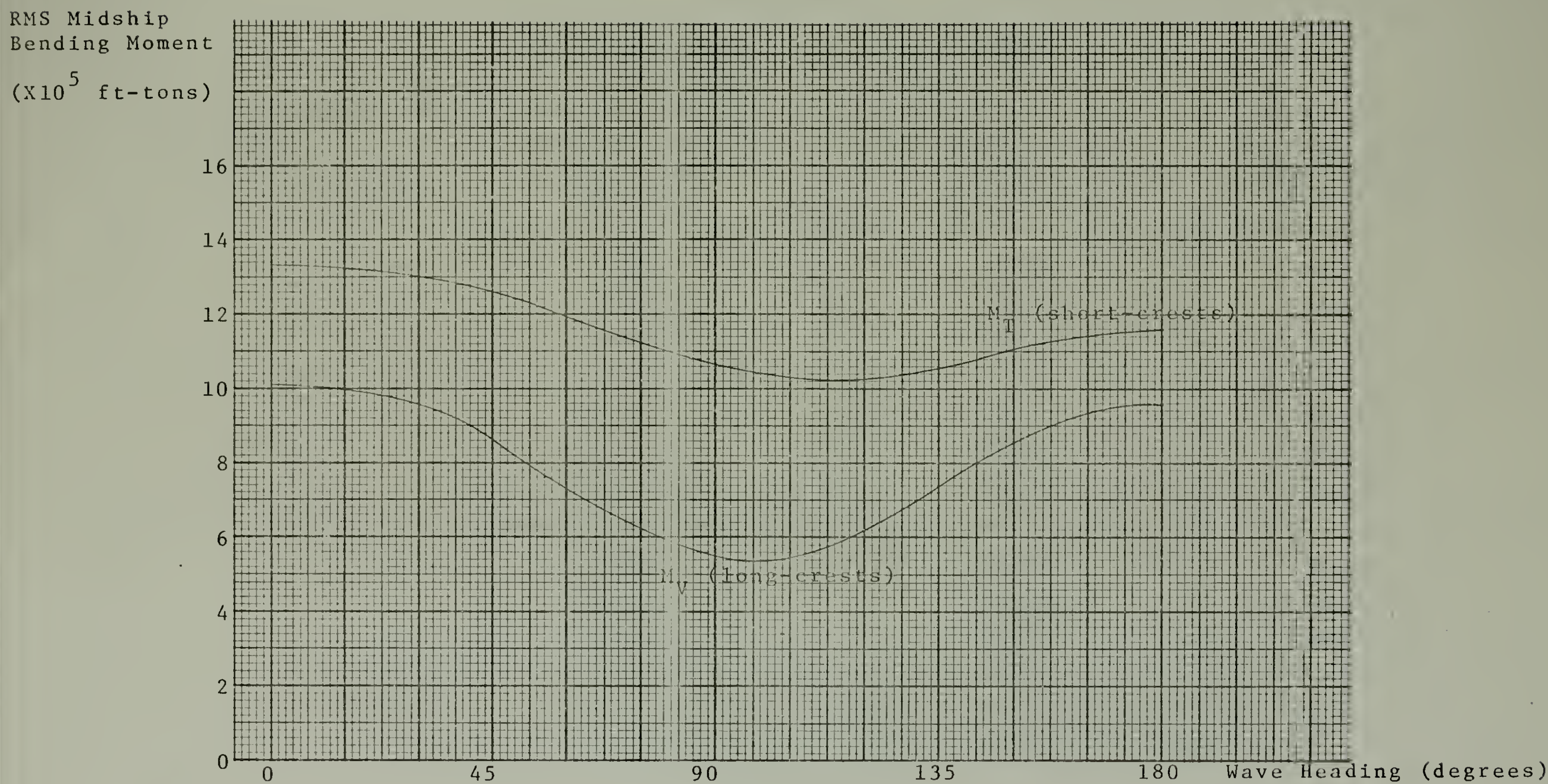


Figure III.11

Comparison of Vertical Moment Response in Long-Crested Seas VS Combined Moment in Short-Crested Seas for h=43.3 ft

Table III.5

The Effect of Speed on the Bending Moments
Experienced in Irregular Seas

Significant wave height (ft.)	speed (knots)	RMS Single Amplitude Vertical Bending Moment M_V(ft-tons)	RMS Single Amplitude Horizontal Bending Moment M_H(ft-tons)	RMS Single Amplitude Combined Bending Moment M_T(ft-tons)
h = 7.4	16	76668.6	112629.9	163295.8
	12	72738.8	110631.4	159227.2
	6	70636.6	110136.3	159172.3
h = 19.2	16	473399.7	490273.9	829528.0
	12	463361.7	506827.1	828541.7
	6	453427.6	510401.6	820420.8
h = 43.3	16	1307839.4	811011.7	1651275.8
	12	1146911.1	845107.0	1657771.0
	6	1133966.6	870121.2	1662724.8

In order to illustrate the procedure to obtain the parameter k for the long-term distribution of the wave response, a "short-term" analysis was done for the worst condition in the ship's route (area 7 in [24]). This is the North Atlantic in the months of December to February and the tabulated data of [24] were used. The results obtained are given as tables III.6 to III.9 for the combined moment. For this analysis, the result is

$$\text{mean RMS} = 367820 \text{ ft-tons}$$

Since this is a "short-term" analysis, one can then say $\bar{m} = 1.25$ (mean RMS) = 459775 ft.tons.

The equivalent values for the vertical and horizontal moments were also computed. These are:

$$\text{mean RMS response, } M_V = 197307 \text{ ft-tons}$$

$$\text{mean RMS response, } M_H = 231043 \text{ ft-tons}$$

Note that the expected value for the horizontal moment is larger because the ship encounters more low-to-moderate seas in any length of time than severe seas.

The mean value of the RMS of the response was also calculated for all the areas in the route of the ship, i.e., a "long-term" analysis was conducted. Both M_V and M_T were calculated and the results are summarized in table III.10.

Table III.6

Mean RMS of the Combined Moment Response in Short-Crested Seas Averaged Over Uniformly Distributed Wind-Wave Directions

Velocity (knots)	Significant Wave Height (ft)	RMS M_V (ft-tons)	RMS M_H (ft-tons)	RMS M_T (ft-tons)
	3.6	13386.3	21411.2	26762.9
	7.4	54212.8	79641.4	115467.5
	10.7	115913.9	159110.8	240620.1
	14.6	212240.4	253534.2	405516.0
V=16	19.2	334744.1	346676.0	586564.9
	24.3	463645.8	426305.6	756493.1
	30.0	591299.8	488621.4	911865.7
	36.4	712415.3	537815.8	1050905.6
	43.3	924782.1	573471.8	1,167,628.3
	3.6	11689.7	19464.6	24197.6
	7.4	51434.1	78228.2	112,590.6
	10.7	112157.7	160,233.2	238895.4
	14.6	206043.0	258953.5	403795.1
V=12	19.2	327646.2	358380.9	585867.4
	24.3	456173.6	442364.9	757601.8
	30.0	583590.6	509395.5	914721.5
	36.4	705103.0	561731.7	1055359.7
	43.3	810988.6	597580.8	1172221.1
	3.6	9831.6	17268.6	22238.0
	7.4	49947.6	77878.1	112551.8
	10.7	110104.1	157927.6	236043.6
	14.6	201175.2	258401.3	398461.4
V=6	19.2	320621.7	360908.4	580125.1
	24.3	449758.8	449322.7	756008.3
	30.0	696331.0	518255.7	913820.7
	36.4	696331.0	573820.7	1056466.6
	43.3	801856.7	615268.6	1175724.0

Table III.7

The Response Matrix R

Wave Period Code / Wave Height Code	x	2	3	4	5	6	7	8	9
00	53154.2	111566.6	160788.2	198591.7	218434.3	222589.5	217320.0	206427.9	192807.0
01	57986.4	121709.0	175405.3	216645.5	238292.0	242824.9	237085.1	225194.0	210334.9
02	68394.2	143554.3	206888.3	255530.6	281062.3	286408.9	279638.8	265613.5	248087.3
03	78058.6	163839.1	236122.5	291638.2	320777.6	326879.7	319153.0	303145.8	283143.1
04	88466.4	185684.3	267605.0	330523.3	363548.0	370463.7	361706.7	343565.3	320895.5
05	88724.6	200837.3	294712.1	365075.0	402783.9	411536.8	402477.5	382712.9	357351.5
06	97462.7	220616.8	323736.8	401029.4	442452.0	452066.9	442115.4	420404.3	392545.2
07	106872.8	241917.7	354994.1	439749.5	485171.5	495719.7	484802.4	460995.0	430946.1
08	115610.9	261697.1	384018.8	475703.8	524839.6	536244.9	524440.4	498686.5	465639.8
09	125021.1	282998.1	415276.1	514423.9	567559.1	579892.7	567127.4	539277.2	503540.7
10	133759.1	302777.5	444300.8	550378.3	607277.2	620422.8	606765.3	576968.6	538734.4
11	142497.1	322556.9	473325.5	586332.6	646895.3	660952.9	646403.2	614660.1	573928.1
12	139605.1	343739.2	498559.4	616796.4	682855.6	703118.8	688411.6	655938.0	613657.3
13	147635.5	363511.8	527237.6	652275.9	722134.9	743563.7	728010.5	693669.0	648956.2
14	156283.6	384805.4	558121.8	690484.5	764435.7	787119.7	770655.5	734302.3	686970.4

Table 7 (continued)

Wave Height Code \ Wave Period Code	x	2	3	4	5	6	7	8	9
15	164313.9	404577.9	586800.0	725964.0	803715.0	808897.8	810254.4	772033.3	722269.3
16	172962.1	425871.5	617684.2	764172.6	846015.8	871120.7	852899.3	812666.6	760283.4
17	180992.5	445644.1	646362.4	799652.0	885295.0	911565.0	892498.2	850397.6	795582.3
18	189022.9	465416.8	675040.6	835131.5	924574.4	952010.4	932097.1	888128.5	830881.2
19	197670.9	486710.3	705924.8	873340.1	966875.2	995566.5	974742.1	928761.8	868895.4
90	205701.4	506482.9	734603.0	908819.5	1006154.5	1036011.4	1014341.0	966492.8	904194.2
91	222379.8	547549.1	794165.4	982507.6	1087734.5	1120012.3	1096584.9	1044857.1	977507.3
92	239058.3	588615.3	853727.8	1056195.7	1169314.6	1204013.2	1178828.7	1123221.3	1050820.3
93	255736.8	629681.5	913290.2	1129883.8	1250894.7	1288014.1	1261072.6	1201585.6	11241[illegible].4
94	272415.3	670747.7	972852.6	1203571.8	1332474.8	1372015.0	1343316.5	1279949.9	1197446.4

Table III.8

Probability Matrix $\underline{P}'$ for Area 7

Wave Height Code \ Wave Period Code	x	2	3	4	5	6	7	8	9
00	.0057695	.0073590	.0003532	.0001766	.0002355	.0000589	.0001177	.0000589	.0
01	.0010597	.0175439	.0024138	.0012363	.0004710	.0002355	.0000589	0	0
02	.0028847	.0509832	.0290239	.0062993	.0017073	.0008831	.0002944	.0001177	.0001177
03	.0041799	.0371482	.0720005	.0253150	.0067703	.0021194	.0014129	.0004710	.0002355
04	.0045920	.0125986	.0580478	.0440951	.0167785	.0038855	.0011774	.0004121	.0002355
05	.0041210	.0066525	.0392088	.0496880	.0228423	.0091840	.0032380	.0005887	.0004121
06	.0043565	.0027670	.0197810	.0406217	.0315554	.0093018	.0034734	.0008242	.0001177
07	.0032380	.0018839	.0137172	.0306723	.0251972	.0105970	.0044154	.0011186	.0002944
08	.0021194	.0004121	.0093606	.0184858	.0177793	.0098905	.0052985	.0016484	.0008831
09	.0042977	.0009420	.0062993	.0167785	.0186036	.0105381	.0054162	.0018250	.0007653
10	.0005298	.0001766	.0008831	.0031202	.0031791	.0032380	.0013541	.0004710	0
11	.0005298	.0002944	.0008242	.0024726	.0036501	.0016484	.0018250	.0004121	0
12	.0002944	.0002355	.0016484	.0034146	.0050630	.0040622	.0014718	.0003532	.0000589
13	.0004121	.0001177	.0010597	.0039444	.0074767	.0038267	.0026492	.0007653	.0002355
14	.0003532	0	.0007065	.0022371	.0024138	.0018839	.0010597	.0002944	.0000589
15	.0001766	0	.0010008	.0015895	.0030025	.0024138	.0012363	.0008242	.0000589

Table III.8 (continued)

Wave Height Code \ Wave Period Code	x	2	3	4	5	6	7	8	9
16	.0002944	.0001177	.0002944	.0013541	.0020016	.0018839	.0007653	.0005289	.0001177
17	.0001177	0	.0002355	.0005298	.0015307	.0011186	.0012952	.0005298	0
18	.0000589	0	.0000588	.0004121	.0017073	.0011774	.0005298	.0003532	0
19	.0001177	0	.0001766	.0007653	.0027081	.002767	.0013541	.0010008	.0004121
91	0	0	0	0	.0000588	.0001177	0	0	0
92	0	0	0	0	0	.0001766	0	0	0
94	0	0	0	0	0	.0001766	0	0	0

Table III.9

"Short-Term" Analysis of Area 7 for Months of December-February.
Weighted RMS Values for the Combined Moment Response

Wave Height Code \ Wave Period Code	x	2	3	4	5	6	7	8	9
00	306.7	821.0	56.8	350.7	51.4	13.1	25.6	12.2	0
01	61.4	2135.2	423.4	267.8	112.2	57.2	13.9	0	0
02	197.3	7318.9	6004.7	1609.7	479.9	252.9	82.3	31.3	29.2
03	326.3	6086.3	17000.8	7382.8	2171.8	692.8	450.9	142.8	66.7
04	406.2	2339.4	15533.9	14574.4	6099.8	1439.4	425.9	141.6	75.6
05	365.6	1336.1	11555.3	18139.8	9200.5	3779.6	1303.2	225.3	147.3
06	424.4	610.4	6403.8	16290.5	13961.8	4205.0	1535.6	346.5	46.2
07	346.0	455.7	4869.5	13488.1	12224.9	5253.1	2140.6	515.7	126.9
08	245.0	107.8	3594.6	8793.8	9331.3	5303.7	2778.7	822.0	411.2
09	537.3	266.6	2615.9	8631.3	10558.6	6110.9	3071.7	984.2	385.4
10	708.6	53.5	392.4	1717.3	1930.4	2008.9	821.6	271.8	0
11	75.5	95.0	390.1	1449.8	2361.2	1089.5	1179.7	253.3	0
12	41.1	81.0	821.8	2106.1	3457.3	2856.2	1013.2	231.7	36.1
13	60.8	42.8	558.7	2572.8	5399.2	2845.4	1928.6	530.9	152.8
14	55.2	0	394.3	1544.7	1845.2	1482.9	816.7	216.2	40.5
15	29.0	0	414.6	1153.9	2413.2	1952.5	1001.7	636.3	42.5

Table III.9 (continued)

Wave Height Code \ Wave Period Code	x	2	3	4	5	6	7	8	9
16	50.9	50.1	181.8	1034.8	1693.4	1641.1	652.7	430.6	89.5
17	21.3	0	152.2	423.6	1355.1	1019.7	1156.0	450.5	0
18	11.1	0	39.8	344.2	1578.5	1120.9	493.8	313.7	0
19	23.3	0	124.7	668.4	2618.4	2754.7	1319.9	929.5	358.1
90	0	0	0	0	0	0	0	0	0
91	0	0	0	0	639.6	131.8	0	0	0
92	0	0	0	0	0	212.6	0	0	0
93	0	0	0	0	0	0	0	0	0
94	0	0	0	0	0	242.3	0	0	0

Table III.10

Mean RMS Value of the Response for all Areas in the Route of the UNIVERSE IRELAND

Area	E[RMS]of M_V Response	E[RMS]of M_T Response	Time Factor
7	154254	299294	2
11	126213	246724	2
18	109398	218546	1
20	66961	138544	1
23	110971	220161	1
28	80454	170798	1
29	92992	189706	1
34	108754	203516	1
41	146576	282585	2
42	123456	246480	2
45	161230	260578	2

Time Weighted Average:
$(RMS)_V$ = 124,562 ft-tons

$(RMS)_T$ = 517,450 ft-tons

$\bar{m}_V$ = 155,702 ft-tons

$\bar{m}_T$ = 646,813 ft-tons

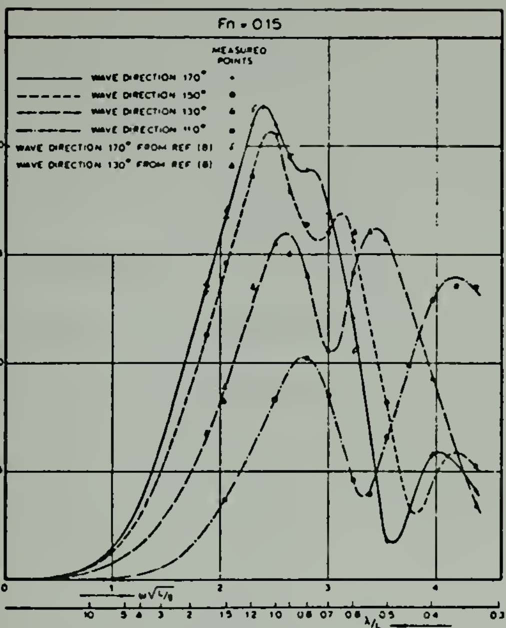
$$\frac{\bar{m}_T}{\bar{m}_V} = 4.15$$

Chapter IV

DISCUSSION OF RESULTS

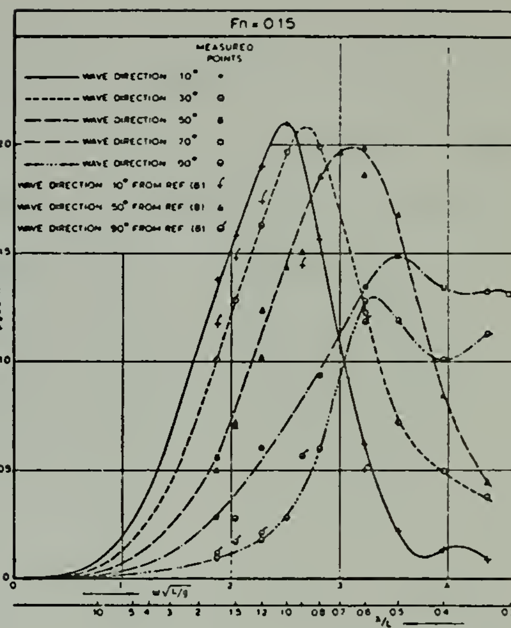
IV.1 Evaluation of the analytical Results

The results obtained in this study agree with the previous findings in other studies that the lateral moment can be very significant for ships in waves. Some definite trends of the moment responses can be observed from figures III.1 to III.4. From the results in short-crested seas shown in figures III.2, 3 and 4, one sees that horizontal and vertical moments change very little with wave direction in moderate seas. On the other hand, there is a broad peak away from the head and following seas for all moment responses in low sea states. In severe seas, the maximum response for M_V and M_T are in the head and following seas while M_H stays almost constant regardless of wave heading. For the long crested seas of figure III.1, one can see that the vertical moment peaks at about 60^o for low to moderate sea states and assumes extreme values at head and following seas in high seas. The equivalent combined moment has two peaks in all cases. The peaks occur in wave headings near 60^o and 120^o with a shift away from the beam seas case as the sea grows more severe.



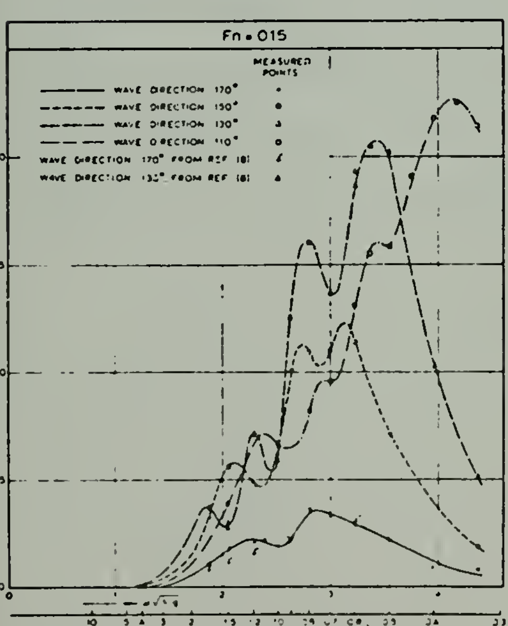
Vertical wave bending moment amplitude in regular bow waves

Figure IV.1a



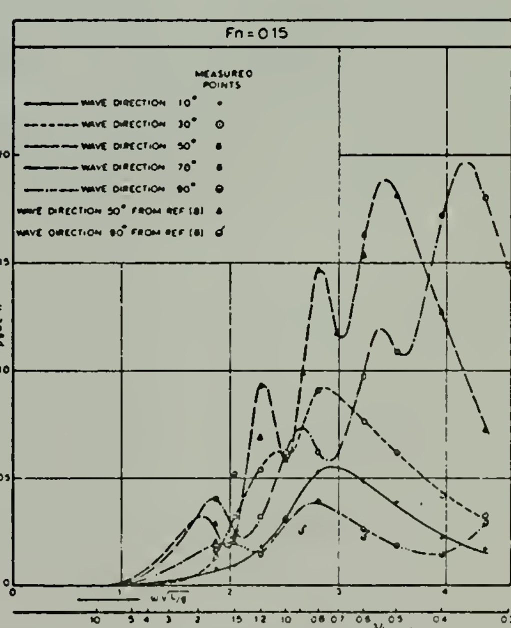
Vertical wave bending moment amplitude in regular beam and quartering waves

Figure IV.1b



Horizontal wave bending moment amplitude in regular bow waves

Figure IV.2a



Horizontal wave bending moment amplitude in regular beam and quartering waves

Figure IV.2b

One observes from figure III.2 and III.3 that the horizontal moments exceed the vertical moments in almost all headings for low and moderate seas. However, in severe seas, for which structural design is based, one sees the customary relation of the vertical moment exceeding the horizontal moment, especially in head and following seas. The reason for this phenomenon is that lateral bending is more responsive to higher frequency excitation than vertical bending. One can see from figures IV.1 and IV.2 that the spectral peak of the lateral moment response for all wave headings are generally situated at higher frequencies than the corresponding response curves for the vertical moments. One can note further that the spectral ordinates of the RAO for wave headings 70^{o}, 110^{o} and 130^{o} are higher for the lateral moment than the vertical moment. Therefore, when the energy of the exciting waves are concentrated in the higher frequency range (i.e., in low sea states), the lateral moment exceed the vertical moment. In high sea states, the reverse trend is true.

For a similar reason the peaks of the response curves for the vertical moment occur away from head and following seas in low sea states. The shift in the location of the spectral peak of the RAO's in the direction of higher circular frequency, ω, can be explained by considering the effective wavelength in oblique seas. It is generally known that the wavelength nearly equal to the ship length causes

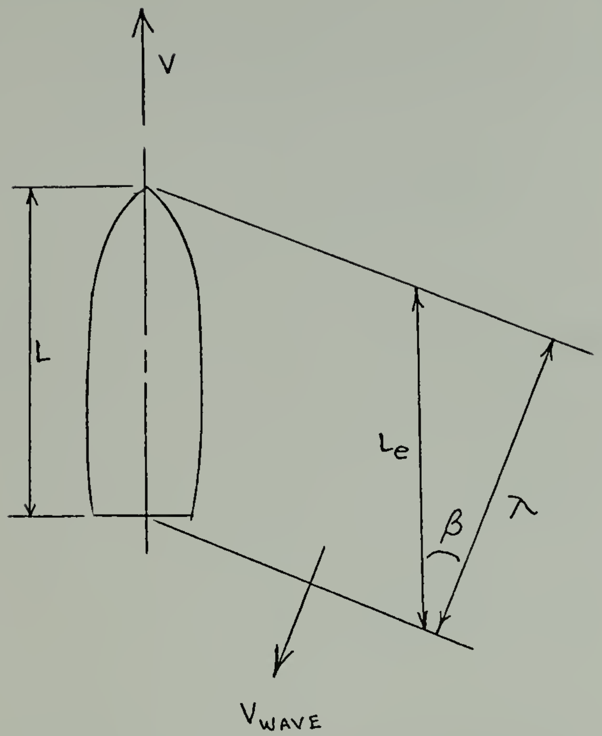


Figure IV.3

The Effective Wavelength in Oblique Waves

the maximum midship bending moments. In oblique seas the ship senses the effective wavelength, L_e, as shown in figure IV.3. The relation with the actual wavelength is

$$L_e = L/\cos \beta$$

where L_e = effective wavelength

L = actual wavelength

β = heading angle

As β approaches 90^o from either 0^o or 180^o, the actual wavelength decreases while the value of L_e stays fixed. Therefore the peak frequency of the response goes up, thereby shifting the RAO spectrum nearer the sea spectral energy density peak in low sea states.

The theoretical calculations predict greater vertical moment response in following seas than in head seas, contrary to experimental results. Wahab's results [15], shown as figure IV.4, confirm this apparent discrepancy between the theory and experiments.

Finally, one notes that the combined moment in "realistic" seas is significantly higher than either of the

components in all cases but, because of the relative phase angles, is not equal to the sum of their individual peaks.

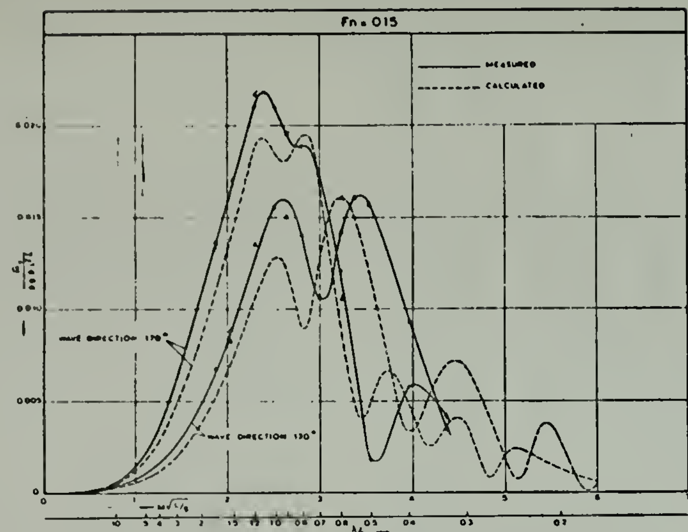


Figure IV.4a Measured and computed vertical wave bending moment amplitude in regular bow waves. See also diagram 1a

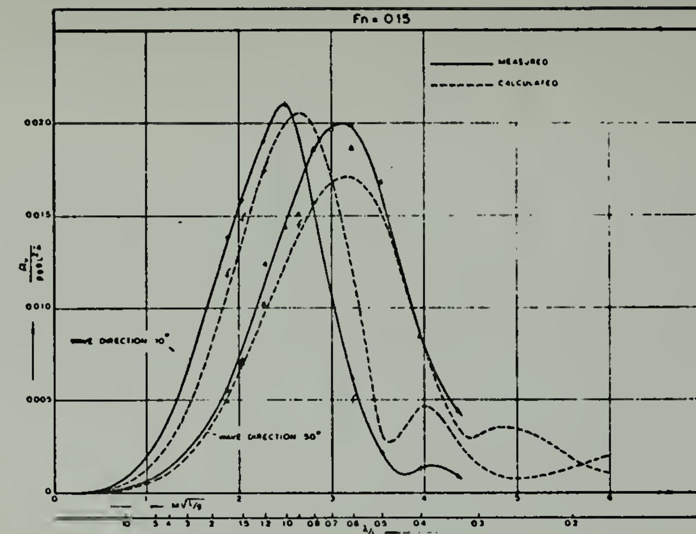


Figure IV.4b Measured and computed vertical wave bending moment amplitude in regular beam and quartering waves. See also diagram 1b

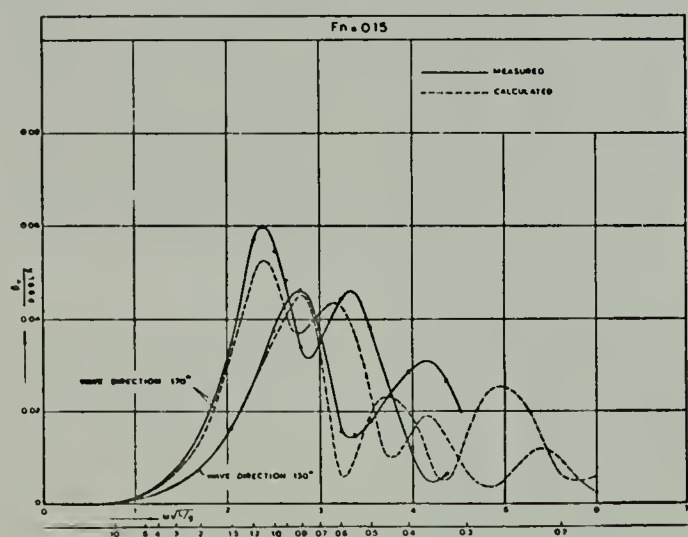


Figure IV.5a Measured and computed vertical wave shear force amplitude in regular bow waves. See also diagram 4a

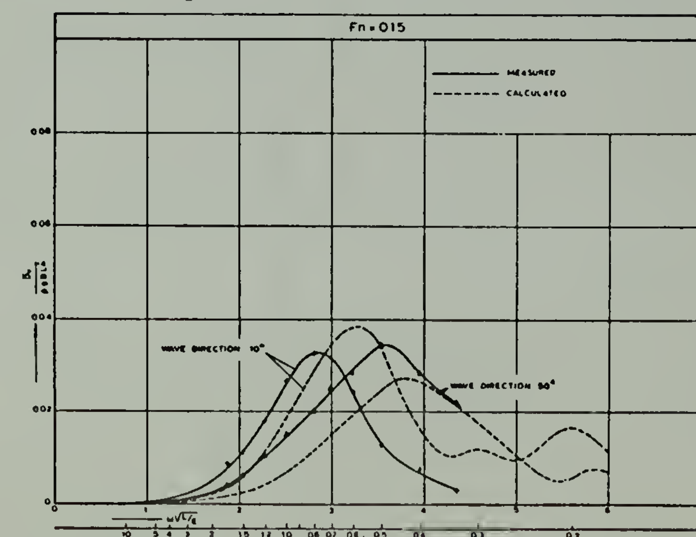


Figure IV.5b Measured and computed vertical wave shear force amplitude in regular beam and quartering waves. See also diagram 4b

IV.2 Comparison of the Values of the Combined Moment

Table III.1 shows the comparison of the response RMS value of the combined moment, M_T, as obtained by the two methods described in chapter two. It can be readily seen that the results are not significantly different, the average for all sea spectra being on the order of eight per cent (8%). This confirms the statement that the average value of the correlation coefficient, ρ_{VH}, already considers the phase difference of the vertical and horizontal bending moment peaks. As would be expected, the third set of values for M_T were closer to the computer values. It is realized that the two values of ρ_{VH} that were used were derived in a different manner and perhaps should not be directly compared. In deriving the second value, it was found that the coefficient varies strongly with the state of the sea and the wave direction and therefore that no single value will be "true" for all conditions at sea. If this method is to be adopted, there is need to study the correlation coefficient further. There is a dearth of published material on the subject.

The calculated value of the combined moment also depends on the relative phase angle of the vertical and horizontal moments. Therefore, it is necessary that the predicted values be at least as accurate as the predictions for the vertical and horizontal moment responses. Comparison of theoretical and experimental phase angles shows that the

agreement is in general quite good [22]. The maximum discrepancies occurred for wave headings of 50° and 110° for the wave lengths $\lambda/L = 1.2$ and $\frac{\lambda}{L} = 1.5$. (Figures IV.6 and IV.7)

In view of the above mentioned considerations it would seem to be indicated that both method 1 and method 2 to obtain the combined moment are equally "good". The first method is probably more convenient for use in analytical studies.

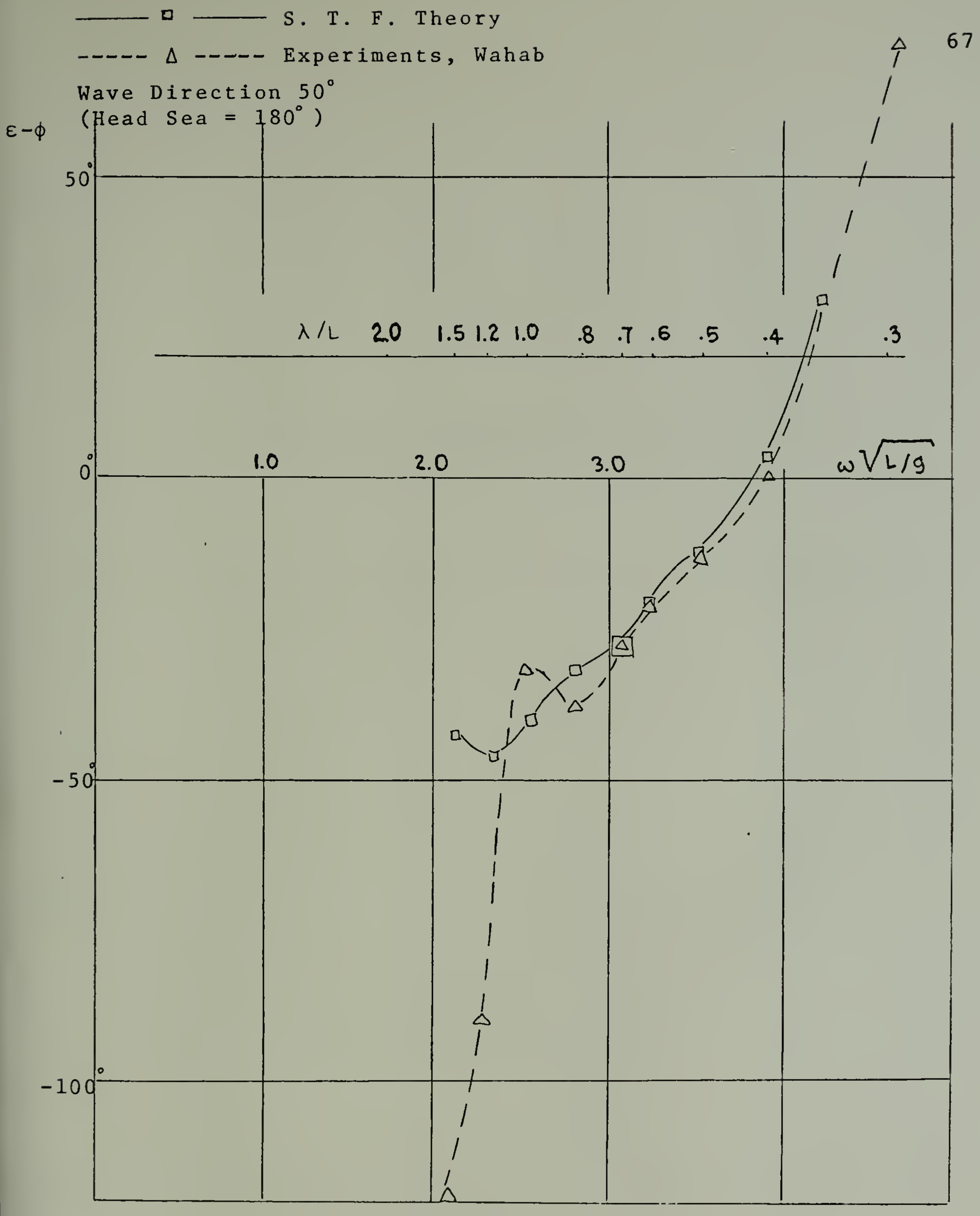


Figure IV.6

Phase Angle Between Horizontal and Vertical Wave Bending Moment at Midship for Series 60, Block 0.80

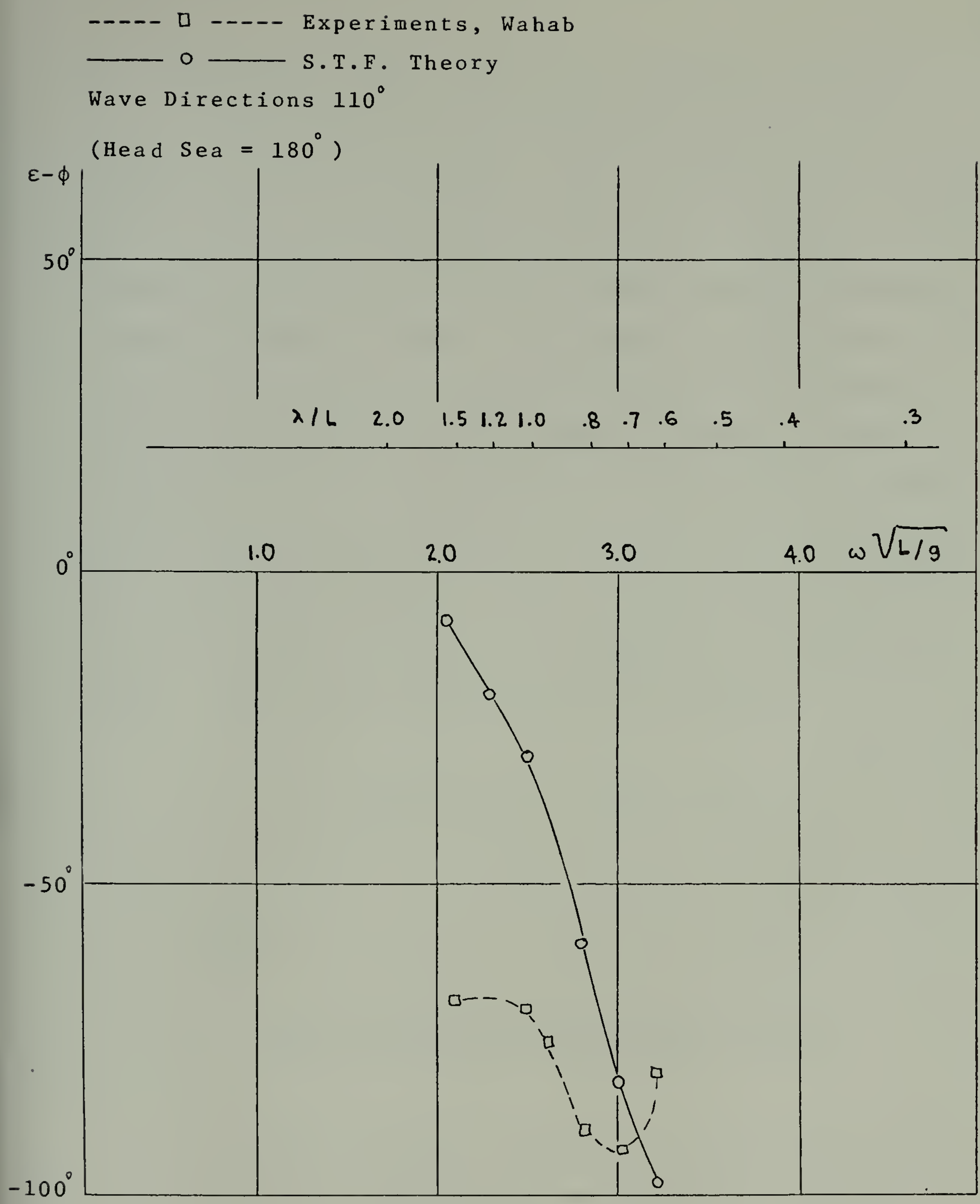


Figure IV.7

Phase Angle Between Horizontal and Vertical
Wave Bending Moment at Midship
for Series 60, Block 0.80

IV.3 Comparison Study of the Vertical Bending Stress

Data on the full-scale measurement of vertical bending stresses on the deck of the UNIVERSE IRELAND can be found in [25]. A comparison between calculated and measured values was done by plotting the average RMS peak-to-peak amidship bending stresses. The analytical values were obtained by calculating the response to each of nine sea spectra corresponding to the observed periods in the tabulated data of [24]. The significant wave heights are the wind-to-wave conversion shown in table III.3. The average RMS peak-to-peak response is the weighted mean of the calculated response to each sea specified by the significant height. This is best described by the relation

$$\text{mean RMS} = \sum_{i=1}^{9} E_V^{\frac{1}{2}} \cdot p(T)$$

where

$E_V^{\frac{1}{2}}$ = RMS peak-to-peak bending stress

$p(T)$ = relative frequency of occurrence of each observed period for a given significant wave height.

The wind-wave relation as described by the Pierson-Moskowitz random and ITTC 1966 line, the corrected Roll curve for the North Atlantic and the corrected Yamanouchi curve for the North Pacific were examined. The conversion is summarized in table III.3.

The use of the Pierson-Moskowitz Random line produced the worst case in the comparison study. While the wind-to-wave relation is intended to convert wind measurements to significant wave heights for seas in random stages of partial development, there is a bias in favor of seas that are near full development [25]. Consequently, the values based on this wind-to-wave conversion, coupled with the known tendency of the theory to over-predict [21], are much higher than the measured stresses. The Roll and Yamanouchi curves were based on observed wave heights and corrected for significant heights. They are believed to be better representation of an actual sea. Since the ship route is neither entirely in the North Atlantic nor in the North Pacific, one sees fairly good agreement with results based on both the Roll and Yamanouchi relations for the lower half of the sea states as described by the beaufort scale. It can be seen that theoretical results over-predicted the average values of the RMS stress, specially in the higher sea states. There are several probable causes of this:

1. Since the largest divergence of compared values are consistently in the more severe seas, one is led to believe that the assumption of linear motions for the ship, in particular resonance in roll, exaggerates the analytical values. Obviously, heave and pitch are large in heavy seas. The divergence caused by roll resonance gets bigger as the energy of the waves are concentrated more and more in the lower frequencies. Faltinsen [22] used a first approximation of roll at resonance as input values. It is also to be noted that the ship was not provided with bilge keels in the computer analysis. It is not known if the acutal ship has these dampeners.

2. Loukakis [21] suggested that deflections of the hull can relieve some bending stresses, so that even if bending moment predictions are accurate the resulting conversion into stresses may result in over-prediction.

3. The inexact weight load distribution used in the analysis could have caused some of the over prediction. A qualitative measure of this effect can be

gained by the excess in value in the calmer seas when the contribution of inertia effects to the dynamic loads are relatively greater.

4. Exaggeration of the reported occurrence of the higher period waves. In high seas, when the ship is pitching and heaving, it is understandable that such errors could be made by untrained observers. In this study, this effect would be very minor.

5. Inaccuracy in the measured data due to insufficient number of records in extreme weather.

For direct comparison of the scatter of the derived values of the stress with the measured values, it was necessary to determine the standard deviations with respect to wind measurements. One can see from table III.4 that the calculated response to the nine sea spectra with the same significant wave height were more widely scattered than the measured RMS peak-to-peak values.

IV.4 Bending Moment Trends and Wave Load Prediction

One can observe from figures III.10 and III.11 that the combined moment in short-crested seas are considerably higher than the vertical moment in head seas. The difference in severe seas is in the order of 15-20 per cent. Therefore, at least for this ship, calculations based on vertical bending only could lead to disastrous results.

Comparison of the horizontal and vertical bending moments in short-crested seas have been partially discussed in section IV.1. It was explained that in low-to-moderate seas the lateral bending response to wave excitation is greater than vertical bending. One consequence of this is its possible significance in fatigue failures. Another is that over any length of time the expected wave bending load will be greater for lateral moments than for vertical moments. This is so because the ship will encounter more low-to-moderate seas than high seas in its lifetime. The result for the short-term analysis for the North Atlantic confirms this statement. The statistics reported in [24] were collected for a period of seven years.

Table III.5 shows that in short-crested seas the bending moment varies only slightly with speed. This statement holds true for all the moment responses, with M_V having the greatest variation for all three sea states shown.

This indicates that reducing speed in a seaway reduces dynamic stresses due to slamming, etc. but will not significantly reduce the primary stresses due to wave bending loads.

Chapter V

CONCLUSIONS AND RECOMMENDATIONS

The subject of the combined effect of horizontal and vertical wave moments has been investigated. A number of conclusions can be drawn from the results of the study.

1. The results obtained by the MIT Seakeeping Program agreed well with published results of theoretical and experimental investigations of ship motions and loads in a seaway. The following bending moment trends were observed:

i. The horizontal moments experienced by the UNIVERSE IRELAND are not negligible in comparison with the vertical moments in almost all conditions at sea. In general, the relative significance of the lateral moments increase with the size of the ships [15]. In low sea states, the lateral moment exceed vertical moments.

ii. The variation with wave headings of the moment response seem to depend on the state of the sea. For long-crested seas, the maximum vertical moment occurs at a wave heading near 60^{o} for low to moderate seas and at 0^{o} and 180^{o} in severe

seas. The horizontal and combined moments, on the other hand, exhibit two peaks near 60° and 120° in all sea conditions. For short-crested waves, all moment responses show a broad peak away from 0° and 180° in low seas. The vertical and horizontal moments flatten out in moderate seas, and the vertical and combined moment assume their biggest values at 0° and 180° in severe seas.

The above behavior of the moments with respect to wave headings can be explained by the character of the RAO.

2. The values of the combined moment obtained by the two methods described in this paper were not very different. Therefore, both methods can be considered valid. The first method involves developing the RAO for the equivalent combined moment by a strip-theory based computational method and performing the usual power spectral analysis. Investigations on the correlation between analytical and experimental values have led to the general acceptance of the method first introduced by St. Denis and Pierson. However, it is felt that further investigations and refinements are necessary before the analytically derived values can be used

in actual design. The second method makes use of the correlation coefficient, ρ_{VH}, to take into account the phase difference of the vertical and horizontal moments. There is also further need to investigate this important statistical parameter.

3. The comparison study of the predicted and measured vertical bending stresses showed good agreement in the lower half of the weather spectra encountered in service. In higher seas, the excessive predicted values are believed to be caused by non-linear motions, contrary to the assumption of the theory. No doubt the addition of bilge keels in the analysis of the ship would have reduced the errors.

   For the ship's route, both the Roll and Yamanouchi corrected wind-to-wave relations seemed to be valid representations of the seaway. In view of the expected higher-than-actual values by theory, Roll's curve is deemed the better approximation of the service route if used in conjuction with the tabulated data of [24], as was done in this study.

4. It was found that there was a significant difference between the values of the vertical moment

response in long-crested head seas and the combined moment response in short-crested seas. Again, it is stated that this may not be true for other ships, particularly the smaller ones. For large tankers with the general characteristics of the UNIVERSE IRELAND, the contribution of the horizontal moment to the longitudinal stresses must be considered in the calculation of the primary strength.

Speed is not a significant factor in the magnitude of the wave loads acting on ships.

It remains to be said that the absolute values of the combined moment, or the total longitudinal stress, as predicted by the methods in this study could not be compared with experimental results. However, the moment trends were studied and found to be reasonable and, on the whole, in conformity with other researchers' results. Therefore, one has confidence on these derived values. The theoretical vertical bending stress predictions could have been closer to measurements but, again, one has firm ideas about the probable reasons for the divergence and therefore one is not completely dissatisfied.

The procedure followed in this study is fairly involved and the cost is not cheap. For further analysis

of ships in related studies, it is recommended that the RAO, or an approximation, be obtained in the cheapest manner. Hopefully, with some anticipation of the expected results, more realistic values can be obtained.

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Appendix A

SAMPLE COMPUTER OUTPUT DATA

I. Irregular Wave Results - Long Crested Unidirectional Seas

Ship Speed = 12 kts.   Heading Angle = 22.5 degrees

Significant Wave Height = 19.2 feet

Peak Spectral Frequency = 0.5197

Motions at Origin:

	RMS	H(1/3)	H(1/10)	H(1/1000)
Heave	0.5974	2.3897	3.0469	4.6003
Pitch	.2456	0.9822	1.2524	1.8908
Sway	.2137	0.8549	1.0901	1.6458
Roll	.4386	1.7546	2.2371	3.3775
Yaw	.1301	0.5204	0.6635	1.0017

Dynamic Loadings:

Vertical B.M.

Sta	RMS	H(1/3)	H(1/10)	H(1/1000)
1	0.0	0.0	0.0	0.0
2	51.1	204.4	260.7	393.6
3	1454.4	5817.9	7417.9	11199.6
4	8894.8	35579.2	45363.5	68490.0
5	41643.6	166574.5	212382.6	320656.0
6	89928.7	359715.0	458636.6	692451.3
7	144749.5	578998.0	738222.5	1114571.0
8	199801.3	799205.5	1018987.0	1538470.0
9	251274.3	1005097.2	1281499.0	1934812.0
10	296087.0	1184348.0	1510043.0	2279869.0
11	329147.6	1316590.0	1678653.0	2534437.0
12	347970.3	1391881.0	1774648.0	2679371.0
13	353323.6	1413294.0	1801950.0	2720591.0
14	344392.0	1377568.0	1756399.0	2651818.0
15	321096.7	1284387.0	1637593.0	2472444.0
16	284297.1	1137188.0	1449915.0	2189088.0
17	180498.0	721992.2	920540.1	1389835.0
18	65363.6	261454.4	333354.4	503299.8

Vertical B.M. (cont.)

Sta.	RMS	H(1/3)	H(1/10)	H(1/1000)
19	24263.0	97052.3	123741.6	186825.7
20	3266.2	13065.0	16657.9	25150.2
21	3990.7	15963.1	20353.0	30729.0
22	3683.9	14735.9	18788.3	28366.7

Horizontal B.M.

Sta.	RMS	H(1/3)	H(1/10)	H(1/1000)
1	0.0	0.0	0.0	0.0
2	4568.3	18273.5	23298.8	35176.6
3	15469.4	61877.6	78894.0	119114.4
4	24915.5	99662.0	127069.1	191849.4
5	52746.4	210985.7	269006.8	406147.5
6	89010.3	356041.5	453952.9	685379.8
7	129957.7	519831.0	662784.5	1000674.6
8	170987.7	683951.0	872037.5	1316605.0
9	208736.5	834946.2	1064556.0	1607271.0
10	238483.0	953932.0	1216263.0	1836319.0
11	258256.3	1033025.5	1317107.0	1988574.0
12	268522.5	1074090.0	1369464.0	2067623.0
13	269648.6	1078594.0	1375208.0	2076294.0
14	260647.3	1042589.5	1329301.0	2006984.0
15	242403.2	969613.0	1236256.0	1866504.0
16	214430.1	857720.5	1093593.0	1651111.0
17	132947.2	531789.0	678031.0	1023693.7
18	48485.1	193940.3	247274.0	373335.2
19	22369.9	89479.8	114086.7	172248.6
20	6595.0	26380.2	33634.7	50781.8
21	456.7	1826.8	2329.2	3516.7
22	118.8	475.5	606.2	915.3

Combined B.M.

Sta.	RMS	H(1/3)	H(1/10)	H(1/1000)
1	0.0	0.0	0.0	0.0
2	4177.5	16710.1	21305.4	32167.0
3	15031.4	60125.7	76660.3	115742.1
4	30139.6	120558.4	153712.0	232075.0
5	86181.8	344727.5	439527.5	663600.3

Combined B.M. (cont.)

Sta.	RMS	H(1/3)	H(1/10)	H(1/1000)
6	164735.4	658941.7	840150.7	1268462.0
7	253106.8	1012427.2	1290844.0	1948922.0
8	342644.4	1370577.0	1747486.0	2638362.0
9	424373.1	1697492.0	2164303.0	3267672.0
10	492267.6	1969070.0	2510565.0	3790460.0
11	540433.3	2161733.0	2756210.0	4161336.0
12	566890.0	2267560.0	2891139.0	4365052.0
13	571413.7	2285655.0	2914210.0	4399885.0
14	554399.9	2213199.0	2821829.0	4260409.0
15	513669.5	2054678.0	2619714.0	3955255.0
16	452582.7	1810331.0	2308172.0	3484887.0
17	281527.8	1126111.0	1435792.0	2167764.0
18	99230.1	396920.7	506073.9	764072.3
19	37856.3	151425.4	193067.4	291494.0
20	6591.3	27805.4	35451.8	53525.4
21	4085.7	16342.9	20837.2	31460.1
22	3671.7	14687.0	18725.9	28272.5

II. Irregular Wave Results - Short-crested Multi-Directional Seas

Ship Speed = 12 kts.

Significant Wave Height = 43.3 feet

Peak Spectral Frequency = 0.3461

Principal Wind Direction = 22.5 degrees

Motions at Origin:

	RMS	H(1/3)	H(1/10)	H(1/1000)
Heave	6.2544	25.0175	31.8974	48.1588
Pitch	1.1184	4.4734	5.7036	8.6113
Sway	3.5923	14.3690	18.3205	27.6603
Roll	4.1180	16.4720	21.0018	31.7086
Yaw	0.6277	2.5109	3.2014	4.8334

Dynamic Loadings:

Vertical B.M.

Sta	RMS	H(1/3)	H(1/10)	H(1/1000)
9	660410.1	2641640.0	3368092.0	5085158.0
10	770714.8	3082859.0	3930646.0	5934504.0
11	852958.4	3411833.0	4350088.0	6567779.0
12	902672.4	3610689.0	4603629.0	6950577.0
13	916354.1	3665416.0	4673406.0	7055927.0
14	891059.9	3564239.0	4544406.0	6861161.0
15	827132.9	3308531.0	4219378.0	6368923.0
16	728960.0	2915840.0	3717696.0	5612991.0
17	456989.5	1827958.0	2330646.0	3518819.0

Horizontal B.M.

Sta	RMS	H(1/3)	H(1/10)	H(1/1000)
9	440864.0	1763456.0	2248406.0	3394653.0
10	518902.5	2075610.0	2646403.0	3995549.0
11	575644.3	2302577.0	2935786.0	4432461.0
12	606.50.0	2424600.0	3091365.0	4667354.0
13	609593.2	2438373.0	3108925.0	4693867.0
14	584602.6	2338410.0	2981473.0	4501440.0
15	535193.7	2140775.0	2729488.0	4120991.0
16	463863.0	1855452.0	2365701.0	3571745.0
17	273484.4	1093937.0	1394770.0	2105830.0

Combined B.M.

Sta	RMS	H(1/3)	H(1/10)	H(1/1000)
9	958354.0	3833416.0	4887606.0	7379326.0
10	1121474.0	4485896.0	5719517.0	8635349.0
11	1242414.0	4969656.0	6336311.0	9566587.0
12	1310208.0	5240832.0	6682061.0	10088601.0
13	1320890.0	5283560.0	6736539.0	10170852.0
14	1273295.0	5093180.0	6493804.0	9804371.0
15	1170845.0	4683380.0	5971309.0	9015506.0
16	1020343.5	4081374.0	5203752.0	7856645.0
17	616199.8	2464799.0	3142619.0	4744738.0

Appendix B

CALCULATIONS FOR AN AVERAGE VALUE OF ρ_{VH}

For comparison with the value of the correlation coefficient of $\rho_{VH} = 0.32$ as obtained from figure III.5, an average of ρ_{VH} was calculated from the moment responses computed by equation (6) of the first method.

$$_{VH} = \frac{E\left[M_T^{\,2}\right] - E\left[M_V^{\,2}\right] - K^2 E\left[M_H^{\,2}\right]}{2K\; E\left[M_V^{\,2}\right]\; E\left[M_H^{\,2}\right]^{\frac{1}{2}}}$$

As the long-term extreme wave moments were not extrapolated, the mean ρ_{VH} value was obtained by averaging over wind/wave directions and for representative sea states. The results of the calculation are summarized below:

V = 16 knots

Short-crested seas responses

wind/wave direction	h=7.4 ft. ρ_{VH}	h=24.3 ft. ρ_{VH}	h=43.3 ft. ρ_{VH}
0°	0.771	.692	.590
22.5°	.799	.700	.598
$45.^{\circ}$	.633	.697	.603
67.5°	.743	.650	.563
90°	.676	.555	.474
112.5°	.587	.441	.367
135°	.460	.384	.334
157.5°	.293	.414	.368
180°	.214	.468	.395
average ρ_{VH}	0.575	0.556	0.477

mean for low, moderate and high seas

$\rho_{VH} = 0.536$

Appendix C

CALCULATION OF THE MEAN RMS PEAK-TO-PEAK VERTICAL BENDING STRESS AND THE STANDARD DEVIATION WITH RESPECT TO WIND

The calculated bending moment was converted into stress by the relation:

$$\text{Stress} = \text{Moment} \div \text{Section Modulus}$$

where the section modulus of the ship at the deck is given as 566,794 in^2 - ft [25].

The computer output gave the RMS of the response. To get the peak-to-peak RMS value, the data was multiplied by the factor $2\sqrt{2}$.

The mean value of the response to a sea described by its significant height is given by:

$$\bar{m}_V = \sum_{i=1}^{9} V_i \, p(T_i)$$

where $\bar{m}_V$ = mean RMS of the peak-to-peak response to a sea specified by the significant height.

V_i = RMS of the peak-to-peak response to the ith sea spectra

$p(T_i)$ = probability of occurrence of the ith sea spectra.

$P(T_i)$ is approximated by the relative frequency of occurrence of the different observed wave periods for a given wave height in the tabulated data of [24].

Average values for all the areas in the ship's route were obtained. The calculation is illustrated in table A.1 for the one case of h = 13.2 feet (wave height code 05 in [24]).

The standard deviation of the vertical bending stress with respect to wave measurements is calculated from the relation

$$S_2^{\,2} = \sum_{i=1}^{9} \left(V_i - \bar{m}_V\right)^2 p(T_i)$$

where s_2 = standard deviation of the bending stress with respect to waves.

For direct comparison with experiments based on wind measurements, the following relation is used:

$$s_1^{\,2} = s_2^{\,2} + \left[s_3^{\,2} - \frac{\overline{\Delta H}^2}{12}\right]\tan^2\theta \qquad [23]$$

Sample Calculations for Table A.1   Wave Height Code = 05 (h=13.2 ft)

Period Code / Area	x	2	3	4	5	6	7	8	9	Row Sum
20	0	22	78	75	32	6	1	1	0	215
23	21	236	1260	1513	660	144	24	10	1	3869
29	1	48	136	127	57	23	5	1	0	398
34	11	103	530	696	408	99	35	21	2	1905
28	3	21	100	142	49	24	4	0	1	344
18	3	292	1425	1571	613	194	86	12	7	4203
7	215	427	2489	3200	1592	456	138	39	20	8576
11	64	409	1921	2420	1235	375	142	43	10	6619
41	16	104	739	864	472	137	39	20	5	2396
42	3	36	241	267	131	28	9	4	0	719
45	31	129	588	860	523	170	40	15	4	2360
Column Sum	368	1827	9507	11735	5772	1656	523	166	50	31604
% Occurence	.0116	.0578	.3008	.3713	.1826	.0584	.0165	.0052	.0016	
RMS Response for h = 13.2 ft	42862	91747	138363	186286	22[illegible]57	247789	256780	255697	247230	
Weighted RMS Response	497	5303	41619	61168	41132	14471	4237	1330	396	

Mean RMS Vertical Moment = 178152.6 ft-tons

Mean RMS Peak-to-Peak Vertical Moment = 503,815 ft-tons

Mean RMS Peak-to-Peak Vertical Bending Stress = 1.99 kpsi

The numerical calculation is illustrated below for the Roll wind-to-wave relation.

$$s_2{}^2 = \sum_{i=1}^{9} \left(V_i - \bar{m}_v\right)^2 p(T_i)$$

Beaufort No	h(ft)	$\bar{m}_v$	$s_2{}^2$(ton-ft)	$s_2{}^2$(kpsi)	s_2(kpsi)
1	7.2	123079.	3.465×10^9	0.573	.239
2	7.6	158320.	5.616×10^9	.878	.296
3	8.0	186316.	6.36×10^9	.0992	.315
4	9.0	229732.	5.188×10^9	.0810	.285
5	10.0	305014.	7.572×10^9	.1184	.344
6	12.4	443627.	1.322×10^{10}	.2064	.454
7	15.0	541361.	2.506×10^{10}	.3912	[illegible]5
8	18.2	803314.	3.279×10^{10}	.5120	.716
9	22.0	964491.	5.530×10^{10}	.8637	.929
10	26.0	1156994.	7.499×10^{10}	1.1712	1.082

$$s_1{}^2 = s_2{}^2 + \left[s_3{}^2 - \frac{\overline{\Delta H}^2}{12}\right]\tan^2\theta$$

Beaufort No	h(ft)	$s_2{}^2$(kpsi)	H(ft)	$\tan\theta\left(\frac{\text{kpsi}}{\text{ft}}\right)$	s_3(kpsi)	s_1(kpsi)
1	7.2	.0573	.8	.115692	2.5	.374
2	7.6	.0878	.4	.115692	2.5	.414
3	8.0	.0992	.6	.115692	2.5	.435
4	9.0	.810	1.0	.115692	2.8	.430
5	10.0	.1184	2.5	.115692	3.4	.516
6	12.4	.2064	2.5	.115692	4.3	.668
7	15.0	.3912	3.5	.115692	5.1	.758
8	18.2	.5120	3.8	.115692	6.1	.997
9	22.0	.8637	4.0	.115692	7.0	1.225
10	26.0	1.1712	4.0	.115692	7.8	1.403

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B205 Baltazar
Combined ship hull horizontal and vertical wave bending moments in irregular seas.